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INTENSITY RELATIONS IN THE SPECTRA OF TITANIUM
II. RELATIVE INTENSITIES OF THE STRONGER MULTIPLETS OF TI I

By George R. Harrison and Harry Engwicht

ABSTRACT

The relative intensities of 16 of the more important multiplets of normal titanium have been measured in the vacuum arc. The intensities of the multiplets in triads are found to agree with Kronig's intensity formulas. The agreement is closest if the effective temperature of the arc is taken as 10,000°A. when the average deviation is about 7%. This is shown to be as good as could be expected, considering the wide separations involved, and the uncertainty of the r^4 and excitation corrections, together with the fact that some of the multiplets contain lines which are abnormal in intensity. A triad due to a two-electron transition is shown to agree with Kronig's formulas for the most closely analogous one-electron transition. Certain definite deviations from the intensity formulas which cannot be due to experimental error are noted. The strongest line in the spectrum under normal excitation conditions is found to be λ3653.49, confirming the selection of this as the raie ultime by spectrochemical means. A number of quintet system lines were found to be stronger fundamentally than this, however, the greater inner quantum numbers overcoming the disadvantage of elevation of the metastable state. Curves showing the relative amounts of self-reversal in multiplets coming from different lower states in the atom are given.

One of us has recently published data on the relative intensities of lines in 34 of the stronger multiplets of normal and singly ionized titanium, Ti I and Ti II.[1] It was found that over 80% of the lines obeyed the intensity formulas deduced from the Sum Rule and the Correspondence Principle by Kronig, by Sommerfeld and Hönl, and by Russell.[2] The exceptional lines were amost equally divided between those abnormally weak and those abnormally strong, with a slight excess of weak lines.

[1] G. R. Harrison, this journal, *17*, p. 389; 1928.

[2] R. de. L. Kronig, ZS. für Phys., *31*, p. 885; *33*, p. 261; 1925. A. Sommerfeld and H. Hönl, Sitz. Preuss. Akad. Wiss., *9*, p. 141; 1925. H. N. Russell, Proc. Nat. Acad. Sci., *11*, pp. 314, 322; 1925.

A general method of photographic photometry was described, adapted from the methods of various workers, which is satisfactory in the spectral range 2500–10,000 Angstroms with a 10 meter concave grating. This same method was used in the present work where the relative intensities of sixteen of the stronger multiplets of Ti I were measured in the vacuum arc. This data gives a means of experimentally testing the formulas of Kronig[3] for the relative intensities of multiplets in triads, and enables us to study the excitation conditions in the arc.

For details of the experimental arrangements and photometric method reference should be made to the first paper. An incandescent tungsten strip at known color temperature was used as a standard of energy distribution in the region 5000–3900 Angstroms, while a Cooper-Hewitt quartz mercury arc running under rigidly defined conditions served in the region 4300–2500 Angstroms. As the energy distribution of the mercury arc must be measured with a spectro-radiometer, and as this distribution varies greatly with the conditions of excitation, it is necessary that the voltage across and current through the arc be kept constant at certain values. It has always been our practice to standardize and use the mercury arc (for energy distribution measurements, but not for plate calibration) at 73.0 ± 1 volts and 3.5 ± 0.1 amperes. As standardization is carried out at frequent time intervals, the true energy distribution of the arc is known with considerable accuracy.

Table 1 contains the values used in this paper for the relative intensities of the principal mercury arc lines. Twenty sets of runs on the spectro-radiometer are averaged, using three different mercury arcs of similar type. One has been in use for three years, one for one year, while the third is new. No differences in energy distribution greater than the variation between runs on a given arc were noted in the spectrum as far down as 3000 Angstroms, which is all we are concerned with here. These arcs were always warmed up very slowly, to prevent rapid devitrification of the quartz.

In the first column of Table 1 are given the wave lengths of the mercury lines used; in the second column are the intensities of the various groups of lines as measured in the focal plane of the spectro-radiometer; in the third the transmission factors of the spectro-radiometer for the appropriate wave lengths; in the fourth the corrected

[3] R. de L. Kronig, ZS. für Phys., *33*, p. 261; 1925.

TABLE I.

Line λ	Measured intensity sum	Transmission factor	Intensity sum corrected	Intensity
4358A.				760
4348	570	.66	870	79
4339				31
4078				
4047	340	.657	520	84
				436
3663				333
3655	1000	.64	1560	387
3650				840
3351				<1
3342	61	.62	113	113
3144				<1
3135				<1
3132	540	.605	890	380
3125				510

intensities just outside of the mercury arc; in the fifth the intensities of the individual lines making up each group, as measured by photographic photometry. The values given are believed accurate within 5%.

A small departure from the standard conditions of current and voltage in the arc would produce only second order errors, provided the lines used were measured or photographed simultaneously. The magnitude of such errors can be estimated from the curves published for the mercury arc by Harrison and Forbes.[4]

THEORETICAL INTENSITY RATIOS OF MULTIPLETS

In Fig. 1 is given an energy level diagram which shows most of the atomic transitions responsible for the multiplets whose intensities were measured. Each arrow represents a complete multiplet. More extensive diagrams for the titanium atom can be found in the papers of Russell[5] on the subject. The strong multiplets measured come from transitions between the middle states of the atom and the low metastable and ground states.

[4] G. R. Harrison and G. S. Forbes, this journal, *10*, p. 1; 1925.

[5] H. N. Russell, Astrophys. Jour., *66*, pp. 335 and 381; 1927.

Kronig[6] has published, in addition to his formulas for the relative intensities of lines in multiplets, formulas for the relative intensities of multiplets originating from the same electron transition and the same parent term. These are deduced from the Correspondence Principle as directly as are the line intensity formulas, and by similar reasoning, so they are presumably as correct as the line intensity for-

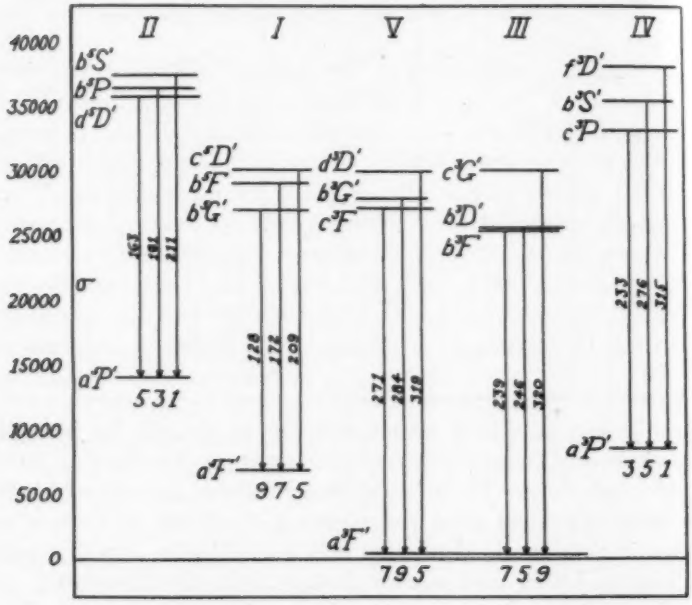


FIG. 1. *Energy level diagram for certain of the lower and middle states of the normal titanium atom. The arrows represent the transitions responsible for the multiplets measured. Each arrow is marked with the multiplet number belonging to it, and below it is marked the relative intensity of that multiplet in its triad, according to Kronig's formulas. The values given for Triad V are questionable.*

mulas which have been confirmed experimentally. This means that they may be expected to be practically exact in certain cases, but to be only approximate in others. As they are not so generally known as the line intensity formulas, we reproduce them here, having substituted Sommerfeld's quantum numbers for Lande's to eliminate the 1/2 in the systems of odd multiplicity considered in the present work.

[6] R. de L. Kronig, ZS. für Phys., *33*, p. 261; 1925.

Considering a transition $(n_1, l_1-n_2, l_2=l_1+1)$, Kronig finds for the relative total intensities of the three multiplets

$$E_{+1}=[(R+\tfrac{1}{2})(a+2)(a+3)(b+1)(b+2)]\div[L_1+1]$$
$$E_0=[-(R+\tfrac{1}{2})(2L_1+1)(a+2)(c+1)(b+1)(d)]\div[L_1(L_1+1)]$$
$$E_{-1}=[(R+\tfrac{1}{2})(c+1)(c+2)(d)(d+1)]\div[L_1]$$

where $a=l_1+L_0+L_1$,
$b=l_1-L_0+L_1$,
$c=l_1+L_0-L_1$,
$d=l_1-L_0-L_1$.

L_0 represents the azimuthal quantum number of the parent term, l that of the electron involved in the transition, L that of the resulting term, and R the multiplicity. The formulas are of course symmetrical with respect to interchange of initial and final states, these being reversed for the cases where Δl is -1 instead of $+1$.

Substituting the values of the various quantum numbers, we arrive at the following multiplet ratios for four of the cases considered in the present paper: $(3d)^3\ 4s-(3d)^3\ 4p$; ${}^5FG:{}^5FF':{}^5FD=9:7:5$; ${}^5PD:{}^5PP':{}^5PS=5:3:1$ $(3d)^3\ (4s)^2-(3d)^3\ 4s4p$; ${}^3FG:{}^3FF':{}^3FD=9:7:5$; ${}^3PD:{}^3PP':{}^3PS=5:3:1$ Kronig apparently does not consider the case of multiplets arising from two-electron transitions, which, however, seem generally to obey the usual multiplet formulas within themselves. In the cases considered above the intensities are given directly by the Sum Rule applied to the groups of levels in a given term as a whole, and they probably afford the least rigorous test of the theory.

It is not sufficient to directly compare the theoretical values given with the experimentally observed multiplet ratios. The ratio of the intensities of two emitted spectral lines can be written

$$\frac{I_{mn}}{I_{pq}}=\frac{e^{-h\nu_m/kT}}{e^{-h\nu_p/kT}}\cdot\frac{\nu_{mn}{}^4}{\nu_{pq}{}^4}\cdot\frac{P_{mn}}{P_{pq}}.$$

The first term on the right is the usual Boltzmann expression for the excitation function, and when multiplied by the á priori probability ratio gives the relative numbers of atoms in the two upper states m and p. The second term, referred to as the ν^4 correction, arises from the expression for the intensity of radiation from a classical oscillator in terms of frequency. The third term contains the usual ratios given by the Sum Rule and the multiplet intensity formulas, with one or

more probable additional factors as yet undefined. In the case of two close lines with a common lower state the whole expression reduces to the radio of the á priori probabilities of the two upper states.

In comparing measured and theoretical multiplet ratios, therefore, we should apply both ν^4 and excitation corrections when needed. It appears that neither the ν^4 correction nor the formulas for multiplet intensity are exact for wide separations between lines.[7] In addition, the formulas may be expected to fail when perturbations are present in the atom, giving rise to inter-system combinations. The titanium spectrum is fairly rich in these, though they are generally of only moderate intensity. Ornstein and Burger[8] have shown that the Sum Rule holds in such cases only when the inter-system lines have been added in with the multiplets from which they have abstracted energy. The results of the first paper indicate the magnitude of the correction to be applied on this account in each multiplet considered here, but we have no basis for estimating the magnitude of the inexactness of the other corrections. We are led to expect only moderate agreement between theory and experiment except in certain simple cases.

EXPERIMENTAL RESULTS

Six plates were exposed to the spectrum simultaneously in the second order of the 10 meter grating, each covering 180 Angstroms in a region rich in strong multiplets. It was thus possible to work on 12 multiplets photographed under identical source conditions. The settings of the cassettes were changed several times during the course of the work to photograph various groups of multiplets together. Eighteen sets of plates were taken, each plate containing ten exposures to the titanium spectrum in addition to the necessary calibration and standardization exposures. Of the resulting spectra only those containing the most accurately measureable sets of lines were reduced. Most of the final intensity values given are the averages of from 25 to 86 separate determinations.

In the first paper a method of correcting for self-reversal in a multiplet was discussed. The method used there can be directly applied to groups of multiplets all having close lower states, or to any group of lines having amounts of self-reversal proportional to their intensities. When multiplets with widely separated lower states are being compared they may have different amounts of self-reversal, but the

[7] L. S. Ornstein, Phys. Zeits. *28*, p. 688; 1927.
[8] L. S. Ornstein and H. C. Burger, ZS. für Phys., *46*, p. 303; 1926.

method can be extended so that this can be compensated for, provided a sufficient number of lines can be measured in each multiplet to accurately get its intensity curve. An example of this is given in Fig. 2 which gives data for eleven multiplets photographed simultaneously. The logarithms of the measured intensities I_m of the lines in the multiplet $a^5F' - b^5G,'$ (128), were first plotted against the logarithms of

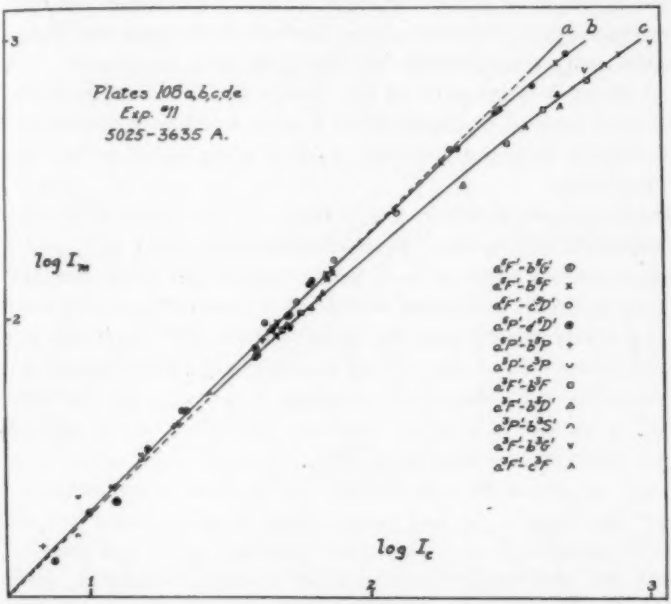


FIG. 2. *Typical curves showing the greater amount of self-reversal in multiplets originating in the ground level of the atom as compared with those originating from metastable states.*

the calculated intensities I_c, the intensity of the strongest line being arbitrarily set at 500. The strongest line of $a^5F' - b^5F$, (172), was then set to its proper log I_m value on the curve, and a similar curve for this multiplet was drawn. In general the two curves were found to coincide. In cases where they did not, one was moved bodily to one side until the closest agreement along their length was found. In this way the relative intensities of the lines in the two multiplets could be determined very accurately, since the actual intensities in each are known from the first paper. Where a multiplet contains six to ten lines the ratio between multiplets can generally be found to a few per cent, even though individual lines may be off much more.

The fact that the curves for the two multiplets fall so closely on one another when they are put together at one point shows that they have similar amounts of self-reversal, and furnishes a very good check on the photometry. It should be remembered that the two multiplets were measured on separate plates, and numerous reduction processes had to be applied to each before the final values were reached, each process introducing a possible error. The comparing of two spectral lines close together is a very simple and accurate procedure compared with the same measurement on two lines 2000 Angstroms, or even 200, apart, as in some parts of the present work. That the curves for 12 different multiplets should fall in a small number of distinct groups in accordance with the positions of their lower states in the atom is very gratifying.

In the example shown in Fig. 2 there is very little self-reversal in the multiplets arising from the metastable states a^5P', a^5F', and a^3P', so they lie on the same curve b, which departs but little from the 45° line a on which all multiplets would fall if there were no self-reversal. The multiplets arising from the ground state a^3F' show much more reversal, however, and they all fall on a curve c, which departs from b. By comparing the values of I_e obtained in this way for the different multiplets we can accurately eliminate the effect of self-reversal so long as it remains less than about 20%.*

In the present work self-reversal was kept to a minimum in the manner discussed in the first paper, most curves showing less of this than the example given. Very pure titanium metal was used in the vacuum arc, this having been obtained from Dr. Augustus Jones of Troy, N. Y. The arc currents used varied from one to three amperes, the average value being about 1.9 amperes. The potential across the arc varied in an inverse manner with the current, between the limits 30 and 90 volts. Under small currents and high voltages the arc was extremely unstable, though these were the best conditions for high excitation and low self-reversal.

The question of excitation will be discussed below, but for the general purpose of the paper it is sufficient to assume it to be constant during the whole investigation. We believe that the photometric errors are less than 10%; the average probable error calculated in the orthodox fashion from the variation of the quantities measured is less than 1%, due to the large number of readings taken. Much of the variation is, we

* The curves for various amounts of self-reversal are drawn tangent to the 45° line at small intensities. The justification for this is found in Paper I.

believe, due to changing excitation conditions, but for the present discussion we will not attempt to separate true experimental error from this.

In Table 2 are given the ratios of the intensities of the strongest lines of a number of multiplets as found from the various spectra on a typical plate, giving an indication of the amount of variation in the readings. All self-reversal and other corrections have been applied. From these values can be found the relative total intensities of the multiplets by multiplying by the proper factor in each case. The numbers in parenthesis are the multiplet numbers assigned by Russell.[9]

TABLE 2.

Plate exposure	^3FG (284) / $^3FF'$ (271)	$^3PP'$ (233) / ^3PS (276)	$^3FF'$ (239) / $^5FF'$ (172)	$^5FF'$ (172) / $^3FF'$ (271)
116- 1	1.51	—	—	1.05
- 2	1.56	1.88	1.02	1.16
- 3	1.44	1.71	1.02	1.08
- 4	1.80	1.66	1.06	1.13
- 5	1.68	1.85	1.02	1.01
- 8	1.76	1.75	1.18	1.04
-11	—	—	0.98	1.62
-12	—	—	1.03	—
-13	1.84	1.81	0.94	1.50
Average	1.65	1.78	1.03	1.20

Table 3 gives a summary of the relative intensities of the various multiplets studied, obtained by averaging together the averages of each plate. In column 1 are listed the multiplet types, while the numbers of the individual multiplets (from Russell's list) are given in column 2. In column 3 are found the final grand average values of the relative intensities of the multiplets for all the spectra measured. Column 4 gives the magnitude of the ν^4 correction, while column 5 gives a number proportional to the measured intensity divided by this; i.e. the numbers in this column represent the relative intensities of the multiplets after the ν^4 correction has been applied. Column 6 contains the wave-number separations of the various upper terms from the lowest upper term, b^3F, the sub-term having the highest inner quantum number having been used for each term. Finally, columns

[9] H. N. Russell, Astrophys. Jour., *66*, p. 381; 1927.

7, 8 and 9 give the relative intensities of the multiplets for infinite temperature excitation, on the basis that the actual temperature excitation in the arc was 12,500°A., 10,000°A., and 7,500°A., respectively. The middle temperature is considered the most probable, for reasons discussed below, and will be taken as the effective temperature of the source.

TABLE 3.

Triad	Multiplet	Number	I_m	ν^4	kI_m/ν^4	$\Delta\sigma$	$I\infty$ 12500°	$I\infty$ 10000°	$I\infty$ 7500°
Column	1	2	3	4	5	6	7	8	9
	^5FG	128	300	163	1840	1522	424	441	477
I	$^5FF'$	172	258	238	1080	3508	314	346	410
	^5FD	209	182	289	630	4672	209	238	298
	^5PD	163	31.8	217	146	10369	93	126	207
II	$^5PP'$	181	22.5	247	91	11026	62	86	146
	^5PS	211	8.8	294	30	11971	23	33	58
	^3FG*	284	245	564	435	2362	110	118	133
III	$^3FF'$	239	202	391	517	0	100	100	100
	^3FD	246	138	407	340	255	68	69	69
	^3FG*	320	191	778	246	4651	81	93	116
V	$^3FF'$	271	157	501	314	1637	73	77	83
	^3FD	319	103	757	136	4524	44	50	63
	^3PD	315	5.8	767	7.6	12771	6.4	9.3	17
IV	$^3PP'$	233	15.8	372	42.5	7726	20	25	36
	^3PS	276	7.8	515	15.1	10051	9.5	12.3	20
	^3FG	347	140	947	148	6240	59	70	95

* Note added to proof, March 18, 1929: In the paper on titanium by H. N. Russell, from which all the configurations used in the present article were taken, multiplet 284 is assigned to the triad marked V in the above table, while 320 is assigned to Triad III. In a recent conversation Professor Russell has informed us that he has recently obtained further evidence which leads him to interchange the configuration assignments originally made for these two multiplets. The above table was first filled out to correspond with the assignments of his paper, but we find on interchanging these two multiplets that the agreement with Kronig's formulas is much improved. As the evidence for term assignments is largely from intensity data, our results appear to be strong evidence for the correctness of the interchange. Table 3 has accordingly been revised in proof with this in mind.

The multiplet intensities given are on the basis that the line intensities in each multiplet are normal. This is not always the case, but

the total intensity deviation is less than 5% in most multiplets and always less than 10%, so it is neglected. The magnitude of the correction to be applied on this account for any multiplet can be readily determined from the tables of the first paper.

A number of interesting facts appear from Table 3. Considering only the next to the last column, we see that some of the quintet multiplets are stronger fundmentally than any triplet multiplet while in a given system the triad of higher L value has an advantage over that of lower L value. While direct comparison does not have very much meaning, due to the fact that the triads represent only a portion of the total radiation due to each electron transition, it is of interest

TABLE 4.

Configuration	Parent term	Triad	Multiplet	Number	I_m	$\frac{kI_m}{\nu^4}$	T_e† °A.	$I\infty$ 10000°	I_e
Column 1	2	3	4	5	6	7	8	9	10
$(3d)^24s-$			^5FG	128	81	118	10,600	89	90
$(3d)^24p$	b^4F'	I	$^5FF'$	172	70	70	—	70	70
			^5FD	209	49	41	8,100	48	50
$(3d)^24s-$			^5PD	163	43	48		44	50
$(3d)^24p$	a^4P'	II	$^5PP'$	181	30	30	—	30	30
			^5PS	211	12	10		11	10
$(3d)^2(4s)^2-$			^3FG	284	85	59	8000	83	90
$(3d)^24s4p$	a^4F'	III	$^3FF'$	239	70	70	—	70	70
			^3FD	246	48	46		49	50
$(3d)^2(4s)^2-$			^3PD*	315	11*	5.3		11.2	50
$(3d)^24s4p$	b^4P'	IV	$^3PP'$	233	30	30	—	30	30
			^3PS*	276	15*	11		15	10
$(3d)^2(4s)^2-$			^3FG	320	86	55	13,300	85	90?
$(3d)^24p$		V	$^3FF'$	271	70	70	—	70	70
			^3FD	319	46	31	8360°	46	50?

* Note added to proof March 18, 1929: Professor Russell informs us that the assignment of the multiplets in this triad to a common configuration is doubtful, and he considers our results as indicating probable incorrect assignment. On account of the large corrections involved, however, further evidence should be obtained before definite conclusions in this regard are drawn.

† Only those temperatures have been computed for which the excitation correction was sufficiently great that a small error in intensity measurement would not affect the apparent temperature greatly. In certain cases, of course, the needed correction is in the wrong direction, and no equivalent temperature can be computed.

to note that Triad I is about 4 times as intense as Triad II, in the same system but with lower L values, while Triad III is about 6 times as intense as Triad IV, the corresponding companion for the triplet system. Again, the two quintet triads added together are about 4 times as strong as the two triplet triads. We thus see the advantage of high R and L values. These ratios would still hold qualitatively if the true temperature excitation in the arc were other than 10,000°A.

In order to compare the results within the various triads with Kronig's formulas, Table 4 gives data in which the center multiplet of each group has been reduced to the theoretical value. This has been done in Column 6 directly for the measured multiplet ratios; in Column 7 after the ν^4 correction has been applied, and in Column 9 after the correction for finite excitation (10,000°A.) is made. In Column 8 is given the excitation temperature which the arc must have had if the multiplet intensities are to obey Kronig's formulas exactly, provided the ν^4 correction is exact. Column 10 gives the theoretical values.

The following conclusions can be drawn from Table 4:

1. Kronig's formulas for multiplet intensities in triads are confirmed, at least qualitatively, in six of the eight cases in which they apply, regardless of the excitation correction.
2. The closest average agreement between theoretical and measured values is obtained if we take approximately 10,000°A as the temperature of excitation of the arc, assuming all levels to be excited at the same equivalent temperature.
3. In Triad V, involving a two-electron transition, the intensities agree qualitatively with the Kronig formulas for the nearest equivalent one-electron transition.
4. In Triad IV there is an exception to the formulas which cannot be explained as due to experimental error. This involves very wide separations between the multiplets concerned.

EXCITATION CONDITIONS IN THE ARC

While the temperature of excitation in the arc necessary to bring the experimental values of the intensities into accord with the theoretical is seen to vary somewhat in column 8 of Table 4, a small experimental error makes a comparatively large variation in apparent temperature. Furthermore, it is reasonable to suppose that all levels are not excited in such a way as to have the same equivalent temperature. We have assumed thermodynamic equilibrium in the computations, whereas

this is far from the truth. The excitation is doubtless of a complex type, and the value 10,000°A chosen as the most probable temperature of excitation may have little meaning except as a basis for comparison.

The source used in the present work was an arc about 3 mm in length at a pressure of 5 cm, most of the light coming from the positive column, although no attempt was made to exclude light from the negative glow. Miss Dewey[10] found in the helium arc that the light from the negative glow originated mainly by recombination of positive ions and electrons, neither having very high temperatures, while in the positive column most of the light came from excitation of neutral atoms by electrons having very high effective temperatures. Nottingham[11] measured the electrical conditions in various arcs, and found in the Cadmium arc electron temperatures of 5400°A for 1.0 and 2.0 amperes, and 6100° A for 4.0 amperes. In the Thallium arc he found 7750°A as the average electron temperature. In the Copper arc,[12] the electron temperature decreased from 19,000°A to 10,000°A with an increase in the arc current from 2.5 to 8.0 amperes. These measurements were all made by the Langmuir probe method. By comparison we see that the electron temperatures indicated in the present work are of the right order of magnitude.

As mentioned above, a source of variation in the results is the change of excitation conditions. This can be studied by comparing the intensities of $^3FF'$ (239) and $^3PP'$ (233), multiplets whose upper states are fairly widely separated, but which are very close together in wave length, and hence can be compared on the same plate with considerable accuracy. The maximum value for the ratio $^3FF'$ to $^3PP'$ was found to be 15.8 and the minimum 9.4, with the average 12.8. Taking the average ratio as corresponding to temperature 10,000,° we find the effective arc temperature to vary between 8000° and 14,000.° An attempt to correlate the temperatures for individual exposures with the arc excitation conditions gave the information that the effective temperature rose as the current decreased, an effect which, though unexpected when found, appears reasonable and is in agreement with some of the results of Nottingham cited above. Any very definite correlation was impossible, since the effective length and cross-section of the arc varied and were unknown.

[10] Jane M. Dewey, Phys. Rev., *32*, p. 918; 1928.

[11] W. B. Nottingham, Jour. Frank. Inst., *206*, p. 43; 1928.

[12] W. B. Nottingham, Bull. Amer. Phys. Soc., *3*, p. 11; 1928.

THE RAIE ULTIME OF TI I

An interesting question has arisen in the literature regarding the raie ultime of normal titanium.[13] Whereas in the neighboring elements in the iron group the raie ultime arises from the transition $d^{n-2}\ s^2 - d^{n-2}\ s\ p$, involving a $4p-4s$ transition, in titanium the line 3653.49 Angstroms, considered the raie ultime,[14] comes from a two-electron transition. Surprise has been expressed that such a transition should be responsible for the strongest line emitted by the atom, and there is also the other question, Why should titanium prefer a different transition than its neighbors on both sides?*

Our results on the relative intensities of the two lines λ3653.49 and λ3371.45 shed some light on this question. We find that under the conditions of excitation used the line 3653 is on the average 1.3 times as strong as 3371, which is the logical candidate for raie ultime. This confirms the selection of the former line spectrochemically. Applying the ν^4 correction, the ratio becomes 1.77:1, showing that the preference is not an accident of position. Finally, 3653 is fundamentally the stronger provided the temperature of the arc was over 5500°A., which seems almost certain. It seems justifiable to consider the two-electron transition as fundamentally a more probable one than the other, as far as these two lines are concerned.

Turning to a comparison of the two multiplets in which these lines occur, we find from the work reported in the first paper that the one near 3370 is somewhat abnormal, while the other is practically normal. There is the possibility, as seen from Table 3, that the whole multiplet containing 3371 is abnormally weak. On the other hand, that containing 3653 is definitely too strong in its triad, and a comparison of the $^3FF'$ and ^3FD multiplets of the two triads indicates that Triad III is stronger than Triad V. We conclude from this that normally the line 3371.45 would be the raie ultime of Ti I, homologous with the raies ultimes of the neighboring elements, but that some perturbation enters in at least one of the triads which pushes the $^3FG-\lambda3650$ multiplet into the lead.

* Note added in proof: The above, and the following paragraphs, were written when the original assignments of these two strong ^3FG lines made by Russell were assumed correct. If we accept his later recommendations and interchange the assignments, as mentioned in previous footnotes and as done in Tables 3 and 4, the whole question of the raie ultime becomes cleared up at once. We leave the paragraphs as originally written to show the force of the intensity evidence for interchanging the assignments.

[13] H. N. Russell, Astrophys. Jour., *66*, p. 30; 1927.

[14] O. Laporte and W. Meggers, this journal, *11*, p. 461; 1925.

When we consider the question of excitation, however, we find that several lines in the quintet system are fundamentally stronger than the raie ultime. The large quantum numbers involved have overcome the elevation of the metastable term to make these lines stronger than any in the triplet system.

In Table 5 is given a list of the first five lines in the titanium spectrum for the normal atom, from the standpoint of intensity. These are as actually measured in the arc, correction having been made for self-reversal. When correction for excitation preferences is made, a number of quintet system lines spring into the lead. The final column of Table 5 shows how the line $a^5F_5' - b^5G_6'$ usurps the place held by $a^3F'_4 - b^3G'_5$ under the lower conditions of excitation.

TABLE 5.

Line	Wave length	King's intensity estimate	Intensity in arc	Intensity under infinite excitation
$a^3F'_4 - b^3G'_5$	3653.49	100 *r*	100	100
$a^5F'_5 - b^5G'_6$	4981.75	60	86	254
$a^3F'_4 - b^3F_4$	3998.65	100 *R*	81	74
$a^3F'_4 - c^3G'_5$	3371.44	80 R	78	89
$a^3F'_3 - b^3G'_4$	3642.68	80 *r*	77	77

The present work indicates strongly the need for controlled excitation in sources used for intensity measurements. We are at present designing a special type of electric furnace for this work, in which we hope to find a minimum of self-reversal.

A portion of the apparatus used in this work was purchased with the grant from the Rumford Fund of the American Academy of Arts and Sciences mentioned in the first paper, and we wish to acknowledge its use.

STANFORD UNIVERSITY,
PALO ALTO, CALIFORNIA,
DECEMBER 27, 1928.

Measurement of Axial Aberrations of Telescopic Systems of Small Magnification.—B. E. Mourashkinsky and M. B. Savostjanova. Trans. Opt. Inst., Leningrad *4*, No. 43, pp. 1928 (Printed in Russian).

The aim of this work is to give a method of measuring longitudinal or axial aberrations (spherical and chromatic) in the case of a telescopic optical system of low magnifying power. The formula is given for the axial aberration, the special advantages of this particular form of expression for the experimental measurements of aberrations being pointed out. By expressing this function in dioptric or similar units, the shape of the curve representing the spherical aberration along the axis is not altered by changing the focusing of the system; which constitutes one of the merits of the method.

The arrangments used in measuring axial aberrations are described. The method of the measurements consists in getting two out-of-focus photographs of a slit or round hole placed at the focus of a collimating object-glass which is illuminated by monochromatic light. The object-glass of the system to be measured is covered by a diaphragm with slit or round-hole aperture. The distance between two photographic plates and the distance between the traces for two definite slits or holes on both the plates having been measured, it is possible to obtain the value of the angle between a given ray and the axis of the system. By means of this angle and the exit-height as measured (in the exit-pupil) we get the distance of the intersection of exit-ray with the axis of the system from the center of the exit-pupil. The values of these distances, as expressed in dioptries, for the series of rays of different entrance- or exit-heights and for different wave lengths being known, the curves of spherical aberration can be traced.

The aberrations of a number of binocular telescopes were measured to test this method. The results of the measurements for the three telescopes are given in the paper. The data for one of these telescopes are given more in detail to characterize the possible accuracy of measurements. The results obtained by measurements with a round hole and round apertures were compared with those obtained by means of a slit diaphragm. Also the results obtained when the refracting power of the system was zero were compared with those when the refracting power was -4 dioptries, and the differences were found to be within the limits of errors. The measurements of the telescopes were compared with the aberration curve obtained by means of computation for the other telescope of the same binocular.

Two telescopes showed so called axial astigmatism and axial coma. The presence of axial astigmatism and axial coma is caused by insufficiently careful adjusting and centering of the optical system of these telescopes.

OPTICAL SOCIETY OF AMERICA

Notice of Election of Officers

In accord with the by-laws, an election will be held in October, 1929 to elect the following officers for the term January 1, 1930 to December 31, 1931:-

President
Vice-president
Four members of the executive council.

The following committee on nominations has been appointed and confirmed in accord with Art. IV, Sec. 3 of the by-laws:

H. E. Ives, Chairman
L. T. Troland
W. B. Rayton
L. B. Tuckerman
H. B. Williams.

It will be the duty of this committee to make nominations for the above mentioned election. The committee's report is to be made not later than August 25. So far as the by-laws are concerned, the committee has full power to decide whom it will nominate; but the committee will be pleased to consider suggestions for nominations from any honorary or regular member of the society who desires to make such suggestions. Address: Dr. H. E. Ives, 463 West Street, New York, N. Y.

Only regular and honorary members are eligible to election. The present incumbents in the offices of president and vice-president are not eligible to re-election.

IRWIN G. PRIEST, *President.*

WASHINGTON
MARCH 25, 1929.

The Physics of Crystals, Abram F. Joffé, Director of the Roentgen Institute and of the Physical Technical Institute, Leningrad, Russia. Edited by Leonard B. Loeb, McGraw-Hill Book Co., New York. 198 pp. $3.00.

This book covers the subject matter of a series of lectures given at the University of California during the year of 1927–28 on the work of Joffé and his collaborators during the last twenty-five years. The subject matter of the book covers two rather distinct fields of investigation of crystals: the first six lectures are devoted to the mechanical behavior of crystals and the last eleven to their electrical behavior. The fundamental mechanical and electrical notions of crystals consisting of regularly ordered atoms and ions and the experimental methods of investigating these are developed not only in the region of small displacements from the equilibrium arrangements, but also when the displacements are large so that first approximations are no longer applicable.

Joffé and his co-workers have taken a subject, which most of us have regarded as rather uninteresting and prosaic during these same years when the structure of gaseous atoms and molecules have been so successfully studied and have largely by their own ingenious, and remarkable experiments made it into one which we must recognize as a most fascinating one. The organization into a consistent system of these searches has brought to us a fascinating account of this work in English. The book should appeal to physicists and physical chemists interested in pure science and also should be very valuable to many interested in purely technical work. Thus, if one wished to study the very important technical problem of electrical insulation and the break down of dielectrics, he could hardly have followed a more profitable course of research than that of Joffé as presented in the last eleven chapters. The investigations of the mechanical behavior of crystals must also have a great technical interest. These researches may completely revolutionize methods of securing higher electric fields and high concentrations of energy.

There are a few typographical errors in the book and the reviewer has noted one error, which may be confusing. Fig. 7 and 8 are not consistent for at the point where the force between the atoms of a crystal is zero, the potential energy must be a minimum.

HAROLD C. UREY

INSTRUMENT SECTION

ON SOME VACUUM RECORDING GAUGES*

K. C. D. Hickman

Large scale, low pressure operations have hitherto fallen into two widely separated classes. On the one hand there is the heavy chemical industry which is concerned with such operations as fractional distillation, the drying of solid chemicals, or the rejuvenation of absorbents, where the residual pressure seldom falls below a centimeter of mercury. On the other hand there is the manufacture of incandescent lamps, x-ray tubes, and radio valves requiring the very highest vacua. Only quite recently has the intermediate region, from 10 mm to 0.01 mm come into prominence for heavy oil distillation and certain drying processes. Whereas the first class of vacuum user has the well known Bourdon tube gauges available for following the progress of evacuation, and the second class has sensitive electrical devices, the intermediate region has been covered by no simple recording manometer whatsoever. An example will illustrate the need.

The most usual substance removed during drying operations is water. Since the latent heat of evaporation of water is large, application of suction involves not only a reduction of pressure, but also a reduction of temperature. Ultimately if no special external heat is applied, an equilibrium is reached where the temperature is low enough to prevent the pump from removing further quantities of water. It is therefore usual to heat internally most vacuum drying ovens, thus keeping the pressure of water vapor high enough to be pumped away. There are certain industrial products of which paper, vegetable and animal fibers, *etc.*, are typical which may be considered as injuriously wet if they are stored when containing but one or two per cent of water calculated as a proportion of their total weight. Removal of this small percentage converts them from wet to dry; from liability to decomposition, to permanency in storage.

If such materials are placed in air-tight containers and evacuated by powerful rotary oil suction pumps, with or without the inclusion of a low temperature condenser in series, the body of the materials will supply enough heat to evaporate the water without the tempera-

* Communication No. 381 from the Kodak Research Laboratories.

ture falling low enough to prevent complete evaporation. The main difference between this and the ordinary industrial drying operation will be the pressure range which, instead of having 1 cm of mercury as its lower limit, will possibly reach a few hundredths of a millimeter.

In this practical illustration, it may be supposed that a thin strip of metal is being manufactured in continuous lengths with a layer of felt glued on to each side, and is then being wound on flat bobbins. It is inconvenient to spread the strip out to dry, but if it is left moist, the metal will rust.

An analysis of a full bobbin might indicate

Non-volatiles	98%
Water, removable at 110°C	2%

It is desired to remove 1 1/2%, or three-quarters of the water by suction. Let the average specific heat of all the non-volatile parts and of the one-half per cent of residual water be 0.2; let its initial temperature be 20°C.

One and a half grams of water in evaporating will absorb $1.5 \times 536 = 800$ calories, approx.

Eight hundred calories extracted from the remaining 98 1/2 grams will reduce its temperature to

$$20° - \left(\frac{800}{98.5 \times 0.2}\right) = \text{approx.} - 20°\text{C}.$$

Therefore, in drying out in this degree, the temperature inside the container, supposing no infiltration of heat, will fall 20° below freezing. At this temperature the vapor pressure of ice is about 0.3 mm, but adsorbed to felt fiber it is probably less than half this amount. In following the exhaustion operation, therefore, a recording gauge would have been needed covering a range from 20 mm to 0.1 mm. Had the water content been greater, perhaps 0.01 mm might have been required for the lower limit.

It was one application of this drying-out principle which led to the development of the series of gauges described below.

The simplest pressure-measuring device, used from earliest times,[1] is the U-tube, half filled with liquid. An increase of gas pressure in one limb drives the liquid into the other causing a relative displacement of the two surfaces directly proportional to the applied pressure. The principle has been used to the limit in two directions. For the

[1] No original reference could be found in 19th Century scientific literature.

measurement of high pressures, Amagat[2] employed mercury columns over 300 meters in length suspended in a mine shaft; for the measurement of pressures less than atmospheric, the mercury U-tube closed at one end, to provide a torricellian vacuum, is well known in laboratories. Instruments of the U-tube class yield a primary or absolute indication of pressure, since their use depends on such constant factors as the specific gravity of the operating fluid and other well defined physical properties of the fluid and containing tube.

The simple mercury U-tube becomes unreliable when read with the unaided eye for pressures less than 1/500 part of an atmosphere, although it is possible to make readings a hundred times smaller with a cathetometer. For very low pressures of the order of a micron[3] and less, recourse must be made to sensitive diaphragms,[4] ionization gauges,[5] viscosity gauges,[6] or hot wire resistance devices.[7] One gauge only, the McLeod,[8] will cover the entire range between atmospheric and a fraction of a micron, but this gauge, invaluable though it has proved through half a century, suffers from certain serious disadvantages. It is rendered useless by condensable vapors;[9] its accuracy is poor when air is the residual gas,[10] or when the glass or mercury are dirty; it is intermittent in action; and it cannot easily be made self-recording.

There have been many attempts to replace the McLeod gauge with extensions of the mercury U-tube manometer principle. Rayleigh's[11] tipping gauge was usable to 0.004 mm, but embodied an optical lever and needed hand adjustment. Hering[12] placed a mercury U-tube in series with a McLeod gauge, measuring the pressure in the limb next to the gauge when both surfaces were brought to the same level. Schrader and Ryder,[13] in their excellent lever manometer, suspended a mirror on a pivoted float, allowing minute variations in level to con-

[2] Comp. rend. *99*, p. 1153; 1884; Annales de chimie, *28*, pp. 456, 464; 1883.

[3] One thousandth of a millimeter of mercury.

[4] Scheel and Heuse. Z. Instrumentenkunde, *29*, p. 14; 1909.

[5] See Found and Dushman, Phys. Rev., *17*, p. 7; 1921.

[6] W. Sutherland, Phil. Mag. *43*, p. 83; 1897; I. Langmuir, Phys. Rev. *1*, p. 337; 1913; 5, p. 212; 1915.

[7] Leithauser and Johansen, Ann. Physik, *24*, p. 25; 1907.

[8] See M. A. Bertin. Annales de chimie 5th series, *19*, p. 231; 1880.

[9] See S. Dushman, High Vacua; Schenectady, p. 88; 1922.

[10] Baly and Ramsay, Phil. Mag., *5*, *38*, p. 301; 1884.

[11] Phil. Trans. 196A, p. 205; 1901.

[12] Ann. Physik, (4), *21*, p. 320; 1906.

[13] Phys. Rev., *13*, p. 321; 1919.

trol the deflection of a light beam. Bleiher and Kohn[14] have substituted oil for mercury in a plain U-tube, while Chattock, Walker, and Dixon have described a tilting manometer[15] using differential columns of liquid. Doubtless all these devices could be made self-recording by the addition of photoelectric cells and dependent electric accessories; or by the use of an optical lever and sensitive paper. It seemed, however, desirable to search for some simpler and more fundamental mechanism.

Such gauges as the pivoted mirror of Schrader and Ryder are able to operate only because in the movements of a considerable area of a mercury surface, even over a minute fraction of a millimeter, quite a lot of work is done. Thus, to depress by one millimeter a mercury surface of 4 sq. cm area, 2.7 gram centimeters of work are involved. If this energy could be utilized by some frictionless lever arrangement, and utilized outside the glass containing walls, a pointer could be moved 1 cm for every mm change of pressure, and the force available would be 2.7 grams just before movement of the pointer. When the pointer had moved to within 0.1 mm of the correct reading there would still

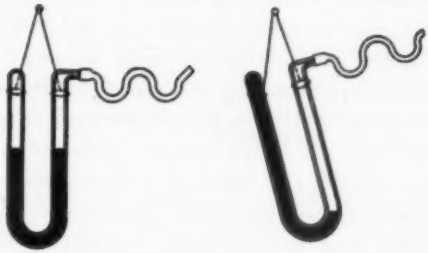


FIG. 1.

be 0.027 or nearly 3 centigrams available for some final movement. The question is how to transmit the movement of a float or other device through the necessarily closed walls of the vacuum system.

When a column of mercury in a U-tube is displaced by differential pressure, the center of gravity of the U-tube, regarded as a separate system, is displaced also. If the U-tube is hanging from a pivot and connected by a flexible pipe to the source of pressure change, the tube will swing with variations of the mercury level. This is suggested by the two extreme positions in Fig. 1. The internal movements of the mercury have now been turned into external movement by an external

[14] Monatshefte Chem., *20*, p. 909; 1899.
[15] Phil. Mag., *1*, p. 79; 1901.

flexible coupling, thus providing an alternative to the lever passing through the walls of the vessel.

The amount of power available may be increased by distorting the shape of the U-tube to accommodate large bulks of mercury. Consider the arrangement in Fig. 2. The limbs *A* and *B* are for convenience equal in diameter and capacity and are connected by a tube C pivoted at *D*. *A* is open to the atmosphere and B is closed and connected by a frictionless coupling E to a source of pressure change. Let the system *A*, *C*, *B* be filled with mercury while in a horizontal position so that it exactly balances D. Let now a slight suction be applied at *E*. Mercury is transferred from *A* to *B* until new levels shown by the dotted lines *a*, *b*, are reached. *B* is now heavier than A and the system tips down at *B's* end. At the same time the levels in the two limbs, as determined

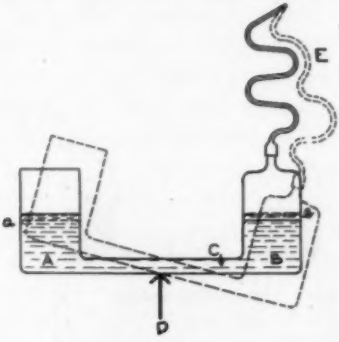


FIG. 2.

by the differences in gas pressure, are disturbed and more mercury must pass from *A* to *B* to restore the relation. *B* is now heavier still, and tips further, taking still more mercury from *A* until a complete upset occurs. The system *A*, *C*, *B* is thus in unstable equilibrium and can be made to tumble over either way by the most trifling change in pressure.

If, however, a spring or other device is attached to provide a counter-turning moment, the tubes will take up a position for each change of pressure dependent on the relative strength of the spring. Since it is a varying column of mercury which is providing the actuating movement, it will be convenient and more fundamental to have a mercury column to furnish the counterpoise. Since also we are dealing at present with a theoretical case, it will be convenient to keep the opposite limbs of the U-system parallel and at a constant distance from the fulcrum

during movement, even though this involves another "infinitely flexible" joint. Ultimately we arrive at the apparatus sketched in Fig. 3. Here the limbs, *A* and *B*, are joined by the flexible coupling, *C*. They are suspended by cords from a large wheel turning on the frictionless pivot, *D*. Below *A* and *B* are two equal plungers, *A'* and *B'*, dipping into the bath, *F*, of mercury.[16] For stable equilibrium the external displacement per unit length of the plungers must be equal to, or greater than, the internal capacity per unit length of *A* and *B*.

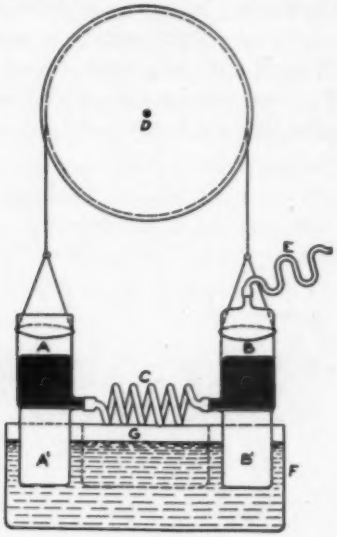


FIG. 3.

We will suppose the displacement of A′ and B′ is greater than the capacity of *A* and *B*. Let now a pressure difference be applied at E so that mercury runs from *A* to *B*. *A* will rise and *B* fall, causing more mercury to transfer by gravity to maintain the new level difference against the sinking of the containers until equilibrium is attained such that, for any deflection:

The increase in turning moment in one direction caused by the pressure transfer of mercury, plus *the increase caused by the induced gravitational transfer of mercury*, is equal to *the increase in turning moment in the opposite sense caused by displacement of the plunger*.

[16] On removal of a central section of the bath it is apparent that the system would remain undisturbed, so that two narrow tubes connected at the bottom would serve equally well as mercury containers.

The more nearly equal A and B are to A' and B', the greater is the deflection necessary to compensate a pressure change. Thus, if A' and B' displace ten units of mass per unit length of run, while A plus B transfer only nine units of mercury, then an applied pressure capable of transferring one mass unit will give a deflection ten units of length instead of one unit of length. Pressure readings in terms of millimeters of mercury will be represented by centimeters of movement. Theoretically there is no limit to the nearness of the capacity and displacements of the tubes and therefore no limit to the magnification of deflection. It is, in effect, a mechanical device for decreasing the apparent specific gravity of mercury to any desired extent and making the deflection obtained in a manometer with this lightened material available outside the apparatus.

Whether such an instrument will be useful or not depends on how many of the ideal considerations have to be sacrificed in practical construction.[16a] The following description of a machine, which is being used in considerable numbers, will show that, between the pressure limits indicated earlier, the principle may be applied excellently.

The first simplification has been to eliminate all flexible couplings except the one connecting the gauge with the gas line; the second, to consolidate A' and B' into one larger plunger. This yields the simple form shown in Fig. 4. Here a broad squat reservoir, A, connects with a taller, narrower limb, B, while a plunger, F, is fastened to the connecting tube, C. The position of E and the fulcrum D are so arranged that when B is full, corresponding to the operating position of the gauge for its most sensitive measurements there is practically no load on the fulcrum.

A laboratory apparatus was made of this pattern and used as a vacuum gauge. When out of use all the mercury would be in the limb A, with the containers balanced as shown in the dotted lines of Fig. 4. When the gauge began to register (at about 7 mm), both containers and plunger were at an angle with the vertical and parallelism was reached only at the end of swing. This destroyed the ability of the gauge to read in absolute units and appeared at first to offer a fundamental objection. It will be shown later that this need not hold, and that the gauge may be used to secure readings directly in, say, grams per square centimeter. Suppose, however, that the compromise construction relegates the gauge to the rank of secondary instruments, will the gauge give a simple relation between deflection and pressure? The model of Fig. 4 was calibrated against a McLeod gauge between

the limits 9 mm and 0.05 mm, and a very nearly linear relationship shown to hold (Table 1). This was considered sufficiently encouraging

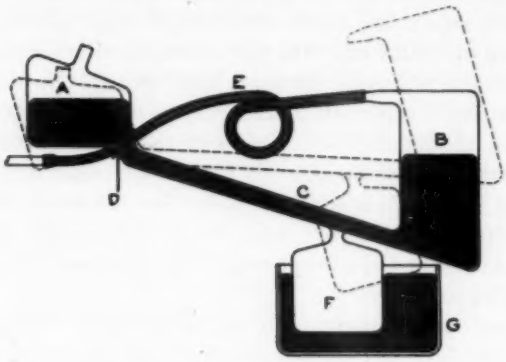


FIG. 4.

to warrant the design of a less cumbersome unit for manufacture in the instrument shop.

TABLE 1

Experimental gauge readings, in centimeters deflection		Pressure in millimeters mercury by McLeod gauge	
Reading	Difference	Reading	Difference
7.0		7.0	
	1.0		1.0
6.0		6.0	
	1.0		1.0
5.0		5.0	
	1.0		1.1
4.0		3.9	
	1.0		1.0
3.0		2.90	
	1.0		1.0
2.0		1.90	
	1.0		0.85
1.0		1.05	
	1.0		1.05
0.0		0.001	

[18a] H. H. Maplesden, U. S. Pat. 387,877—1888; C. Herschel, U. S. Pat. 1076437—1913.

The improved gauge is drawn in Fig. 5 from which it will be seen that the most important changes consist in placing the dash pot on a level with the manometer tubes and housing the latter in a cradle with an adjustable carriage holding the plunger riding on an extension arm. It is much more convenient to vary the leverage of the plunger than to alter its diameter when it is necessary to change the sensitivity of the balance. The glass work has two important modifications. First, the plunger has a bulb at its lower end. This keeps it buoyed above the mercury in the dash pot until the mercury in the tubes has been drawn away from the top rounded portions of the glass work. The cylindrical

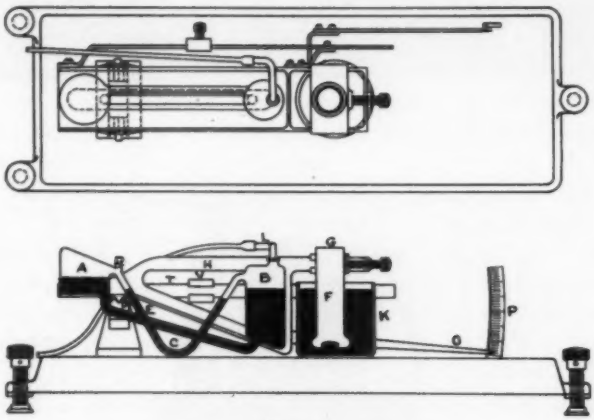


FIG. 5.

part of the plunger is immersed only when the mercury surfaces are traveling in the cylindrical portions of the other tubes. The second alteration is the addition of a short-circuiting channel to take care of accidents. If the vacuum is suddenly broken when these gauges are in operation, the mercury rushes into the closed limb and smashes the top. Any sufficient constriction placed in the communicating tube slows the action too much. The short circuiting tube avoids this calamity. Normally the tube is full of mercury. When the pressure is released slowly, the mercury is not driven out because the inverted part is deeper than that of the normal return tube. A sudden influx of gas, however, scavenges the tube and equalizes the pressure in the two limbs long before any appreciable movement of the main body of mercury has taken place.

To describe Fig. 5 more particularly, *A* is the closed limb, *B* the working limb, and *C* the connecting tube. The fulcrum and pivot *D* are split in the middle to allow *C* to pass. *E* is the safety tube; *F*, the plunger

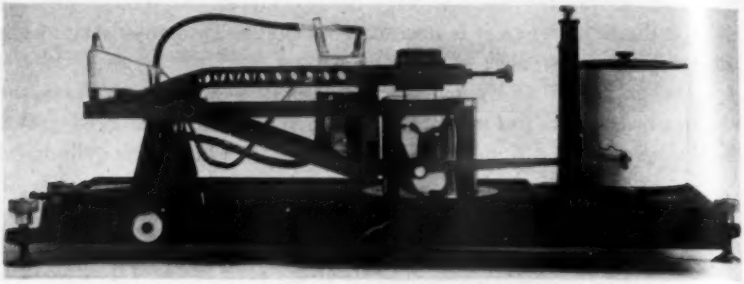


FIG. 6.

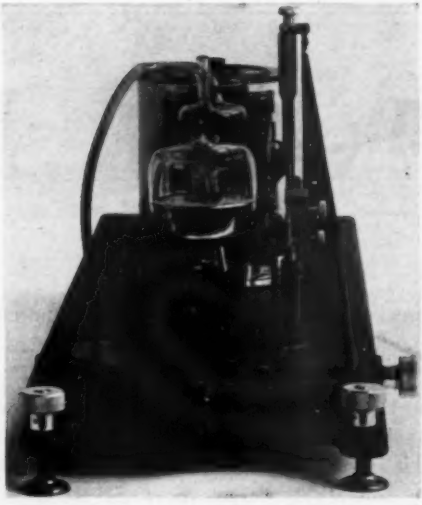


FIG. 7.

fastened to the carriage, *G*, adjustable along an extension of the cradle, *H*, which houses *A* and *B*. The mercury dash pot, *K*, is large enough to allow for the changing slope of *F* during operation. The lead-in tube, *L*, may be of some flexible material, the walls of which will not

collapse under suction.[17] Experience with the gauge soon showed that so much power was available at the pointer O at the scale P, that a recording pen could be attached without impairing accuracy. The scale was therefore replaced by a rotating clock and chart. Safety rests were provided at the fulcrum, D; a rider bar was placed the length of the cradle, for ease in zero adjustment and finally the instrument was covered with a glass case. Working drawings were prepared so that the gauges might be duplicated in small numbers, and a patent application was filed. Photographs of the finished instrument are shown in Figs. 6 and 7.

PERFORMANCE OF THE GAUGE

The gauge should be accurately leveled on a very secure foundation.[18] The range and sensitivity are determined by adjusting the position of the plunger on the extension of the cradle and adding the right quantity of mercury to the dash pot. The instrument is calibrated conveniently by comparison with a McLeod gauge.

The calibration curve (reproduced in Fig. 8) shows one regrettable quality. The readings on the downward and upward journeys are different, disclosing a pronounced hysteresis. The gauge in this condition is therefore quite unsuitable for intermittent fluctuations. Only when the variation is in one direction and gradually increasing, will the record be trustworthy.

The reason for the trouble is soon apparent. It is not easy for any long period to maintain absolutely clean glass or mercury surfaces and under everyday circumstances it is quite impossible. Consequently there is a definite drag when the edge of the liquid meniscus moves up or down the tubes. The trouble is not noticeable in an ordinary barometer tube where the center of a wide mercury column may rise or

[17] The material composing the tube must not be hygroscopic or give off gases which tarnish mercury or appreciably spoil the vacuum. Of a large number of substances tried, the kind of rubber tubing used to connect automobile wind screen wipers has been found the least objectionable. It should be the smallest procurable (1/8-inch internal diameter), of a pure gray color, and should not smell of rubber. It has been recommended to boil such tubing in dilute caustic soda to remove sulfur, but we have found it better to thread through the tube five or six strands of fine copper wire taken from lamp extension cord. A length of the threaded tube about two inches longer than the shortest required is then cut and the end slipped onto the connections so that the tube takes a slightly arched path. Any sulfur subsequently diffusing from the rubber attacks the copper and leaves the mercury untarnished. The copper wire may be made even more efficient by bathing it momentarily in 5% nitric acid containing a little mercurous nitrate, washing, and drying.

[18] See p. 327 relative to leveling precautions.

fall one tenth of a millimeter without moving the boundary of its surface with the glass. In the present differential gauge, however, the movement is multiplied many times and a fluctuation of a tenth of a millimeter may necessitate a movement of two or three millimeters in three tubes, which movement can take place only if the edge of the meniscus will move against the wall.

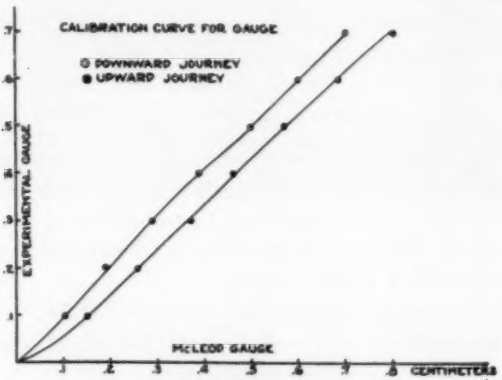


FIG. 8.

It is well known that comparatively dirty mercury which sticks to dry glass will become mobile under water. Water lubricates the mercury against glass in the presence of dirt. It was thought that possibly some organic liquid having the same lubricating properties and a negligibly low vapor pressure might be found. Such a liquid could be placed in the gauge to make the mercury slip easily up and down the tubes. Also, apart from the immediate purpose it would be useful in many experimental instances for preventing mercury from sticking to glass.

Although the perfect liquid has not yet been found, there are a number which work so well that their use is more than worthwhile. The full description of these compounds and their many uses has required a separate communication,[19] and it is not possible even to summarize the results. Two liquids will be recommended here, *viz.*,

n-dibutyl phthalate
tetra-ethylene-glycol methyl ether.

[19] To be published shortly.

The butyl phthalate has a vapor pressure less than one thousandth of a millimeter at 30°C. The glycol exerts one twentieth of a millimeter at the same temperature, but, though it is not suitable for use in high vacua, its lubricant qualities are the best yet found, being about twice as good as the butyl phthalate. Mercury wetted with either of these materials slips easily over glass even though appreciable quantities of grease, mercury sulfide, or metallic dross are present. They improve the mobility of the gauge by at least ten times. The liquid recommended for general use with the gauge is butyl phthalate. About two drops should be introduced into the closed end while four or five may be left in the open. Evacuation and warming should proceed exactly as though no extra liquid were present. The beneficial effect on sensitivity and lag are apparent from the graphs in Fig. 9. There

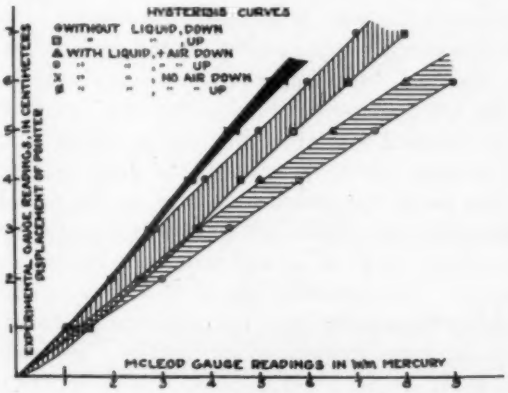


FIG. 9.

is still some hysteresis, but it is due to a different cause—residual air. The solubility of gases in the organic liquids is the great drawback against their use. Placed above barometric columns they may either be allowed to contain dissolved gases, or they may not. If they do, a true barometric reading is not obtained; if they do not, it is sometimes impossible to break the mercury away from the glass to renew the torricellian space after this has once been allowed to disappear.

The design of the present gauge demands that a trace of gas shall be left, *i.e.*, the boiling out must not be too thorough, otherwise the mercury cannot be coaxed out under vacuum. Even vigorous jars fail to start the free space after the gauge has been out of use over

night. A little experience, however, enables just the right quantity of gas to be left behind. Too much residual gas introduces the following inaccuracies (in order of decreasing importance): 1. Hysteresis. 2. Increases temperature coefficient and shifts zero point. 3. Makes the gauge sensitive to barometric variations.

The hysteresis is interesting. As the cradle tips down under diminished pressure the free space in the closed limb increases and dissolved gas evaporates, without, however, keeping entire pace with the increase of space. The pressure of residual gas is, therefore, less than at the start, and the gauge readings are a little too high. On the return journey the liquid which has followed the meniscus down the glass becomes temporarily buried beneath the rising mercury, and is not immediately available for solution of the residual gas. The pressure in the closed limb is now higher than it was at the start and the readings are correspondingly too low. In practice the effect is noticeable only with the upper readings of the gauge.[19] As the pressure gets lower, the discrepancy vanishes. Most important, however, is the fact that the mechanical lag has been *entirely removed* for small quick fluctuations, which may be recorded with an accuracy of a fiftieth of a millimeter.

The effect of temperature variation on the gauge readings was next examined. The gauge was placed in a large double walled box fitted with a double celluloid window. Near one end was a small electric fan playing onto a vessel of ice and salt, or an electric hot plate, as the case may be. Thermometers placed at various positions showed that this crude arrangement kept the temperature to within 3/4°C either side of the mean irrespective of position. Pressure deflection curves were made at $42° \pm 3/4°$C, 22°C (room temperature), and $6° \pm 3/4°$C, the curves being recorded in Fig. 10. There is no evidence of the liberation or suppression of vapor at the high or low temperatures influencing the upper readings. The zero point of the gauge is merely shifted a small amount, evidently caused by the differential expansion of steel, mercury, and glass. The shift is actually 0.0055 mm or about one two hundredth of a millimeter per degree Centigrade, the gauge reading too high on warm days, too low on cold. The error can be

[19] It should be emphasized that the hysteresis curve is obtained by taking the gauge from atmospheric pressure down to a good vacuum and then admitting gas until the pointer has left the scale. There is thus recorded one complete deflection in either direction. When the gauge is in permanent use it becomes matured to vacuum conditions and the hysteresis for one or two centimeters' deflection from the mean reading is not detectable. The hysteresis curves shown in this paper represent abnormal and extreme cases.

almost eliminated by adjusting the zero point before any day's run.

The General Electric Company has a number of these gauges recording the pressure in the backing lines for the mercury pumps in their lamp exhausting machines. The charts have shown a number of irregularities long suspected but not hitherto accurately measured. From these charts checks have been made which have materially increased the efficiency of the backing lines. This example is typical of the field of usefulness for a differential gauge filled with mercury lubricated with an organic liquid.

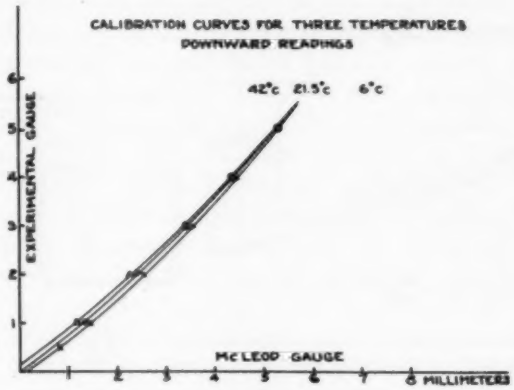


FIG. 10.

The Gauge as an Absolute Manometer.—It was stated earlier that the gauge could be used as an absolute instrument without comparison with another standard. This can be accomplished by employing the null-method of no deflection in conjunction with compensating weights. Reference to Fig. 11 will make this clear.

Let A, B, C, D, E, represent the differential gauge and let F be a balance pan attached to the pointer, G. With a good vacuum applied at L, the gauge takes up a zero position with the cradle horizontal. Let now some gas be introduced so that the cradle swings, disturbing the pointer from G to the new position G'. Note this new position of the pointer. Now without altering the gas pressure, return the cradle and pointer to G by addition of weights W at F. The mercury in A and B will stand at the new levels shown by the dotted lines.

Let the mass of mercury transferred from A to B by the pressure change of p_{mmHg} be M grams.

Let ΔA be the area of A, and
ΔB be the area of B.
The difference in level of the two surfaces will be

$$\frac{M}{\Delta A \cdot S}+\frac{M}{\Delta B \cdot S}=h_{mm} \tag{1}$$

where S is the specific gravity of mercury. The change in turning moment about D will be proportional to the mass transferred and the distances of the centers of gravity of the limbs from the fulcrum. Let

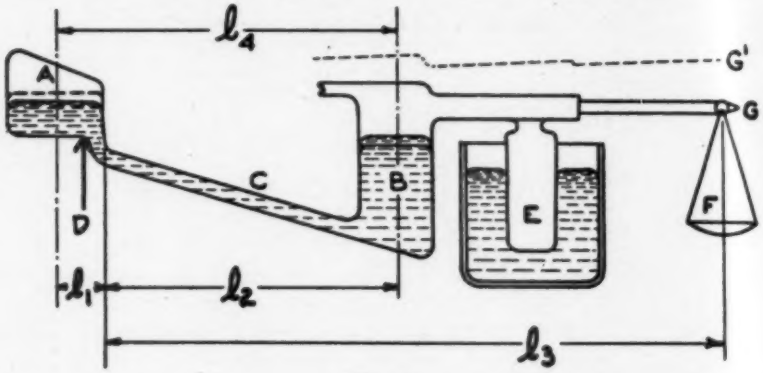


FIG. 11.

these distances be l_1 and l_2, with the distance of the pan, F, l_3. Then: Ml_1+Ml_2, must be equal to Wl_3, where W is the weight in grams added to the pan. That is,

$$W=\frac{M(l_1+l_2)}{l_3} \tag{2}$$

now from (1)

$$M=\frac{h\cdot\Delta A\cdot\Delta B\cdot S}{\Delta A+\Delta B}=h\cdot\frac{r_A^2\cdot r_B^2\cdot S}{(r_A^2+r_B^2)} \tag{3}$$

substituting (3) in (2), we have

$$W=h\cdot s\cdot\frac{(l_1+l_2)}{l_3}\cdot\frac{r_A^2+r_B^2}{(r_A^2+r_B^2)}\cdot \tag{4}$$

$$\text{Whence, } h=p_{mm}\cdot H_g=\frac{W}{S}\cdot\frac{l_3}{(l_1+l_2)}\cdot\frac{(r_A^2+r_B^2)}{r_A^2\cdot r_B^2}\cdot \tag{5}$$

The value of the coefficient

$$K=\frac{l_3}{(l_1+l_2)}\cdot\frac{(r_A^2+r_B^2)}{r_A^2\cdot r_B^2\cdot S}$$

can be determined once for all and denoted as K = constant of the gauge. Then $p=K.W$.

Since the position of the pointer for the pressure p was noted before the weights were placed in F, the deflection corresponding to the calculated pressure is known. In a similar manner it is possible to calibrate the whole scale, *weighing the pressure* at each point.

An interesting application consists in fitting the gauge with two long rider arms each accommodating movable weights, one having ten times the weight of the other. The heavier moves from notch to notch, each step denoting, say, 1 mm. The lighter moves over a scale divided in hundredths of a millimeter. With both weights at zero the gauge is allowed to deflect to the pressure to be read. An approximate reading is secured which may be verified by shifting the weights until the cradle is restored to the null position. In an actual example a pressure of 4.70 mm was recorded by deflection. The weights were then shifted until balance was secured and the pressure found to 4.77(5) mm. The weighing takes practically no time because substantially the correct weight is available for the experimenter before he shifts the riders.

In the calculation of K, the factors W and S, r_A and r_B are known or can be found with considerable accuracy; l_1 and l_2, however, are more difficult to find individually. Since, however, it is only their sum or the over-all length between the centers of the limbs which is involved in the equation, the readings are independent, except for zero adjustments, of the position of the fulcrum. The total distance (l_1+l_2) is found sufficiently accurately by subtracting half the sum of the external diameters A and B from the over-all external length l_4.

A PORTABLE MODIFICATION

The gauge pictured above is essentially a laboratory recording instrument. Its portability is destroyed chiefly by the open mercury dash pot.

Assuming that a spring is not permanently an accurate enough substitute, how can the dash pot be abandoned? Consider the symmetrical U-tube system A, B, C, D, stabilized by the spring E in Fig. 13.

The level of mercury in the two limbs is at a and b. Let a small pressure-change alter these to a' and b', transferring a volume v from limb A to limb B. The spring now relaxes and a further volume v' is transferred by gravity. This quantity is added or subtracted, not to the top

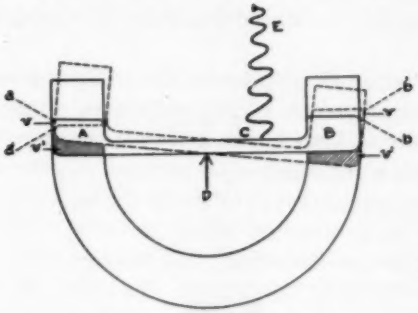


FIG. 13.

of the columns, but to the bottom, at the places shown by the shading; and it results from the movements of the bottoms of the tubes. To relieve the spring E of the burden of this gravitationally transferred mercury it is necessary to provide A and B with virtually no bottoms. This is done by making them parts of a completed semi-circular annulus.

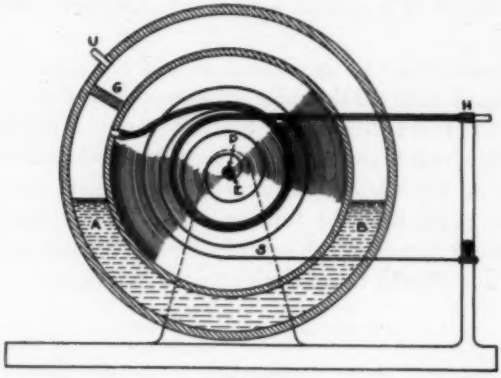


FIG. 14.

Any movement of this circular tube about D results in a mercury transfer, merely that determined by pressure. To preserve the symmetry after deflection the workings portions, A, B, must also be semi-circular in elevation, leading to a construction shown schematically in Fig. 14.

The mercury is contained in a circular channel A, B, turned out of a steel disk, on to the face of which is cemented a glass plate. The disk is pivoted on small ball bearings at D and its position is controlled by the spring S secured at E.[20] A partition C separates the gas space above A and B. The tube G connects the limb A with a coil of flexible lead-in tube at H. The vent U in the side B may be opened to the atmosphere for pressure readings or kept closed, preserving a torricel-

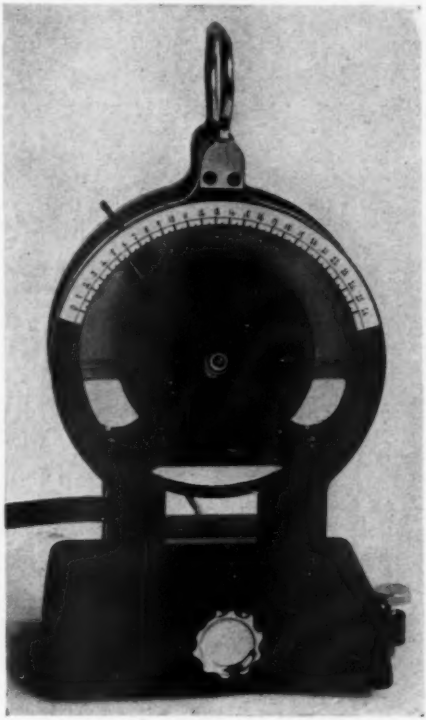


FIG. 15.

ian vacuum, for vacuum readings. Consider U closed and a good vacuum above A and B. The disk is perfectly balanced and may swing freely in either direction unhampered by the mercury which is able

[20] Although this form of gauge is compensated by a spring, nine-tenths of the load has been removed by elimination of the dash pot. A well made clock spring working under small stress remains sufficiently constant for use in this apparatus.

always to flow to the lowest position, maintaining the levels equal in either quadrant. The spring S is secured so that it exerts no turning moment. The lead-in tube H is now connected to a source of slight pressure. Instantly there is a reaction between the mercury surfaces A and the barrier G, equal to the pressure and equal to the weight of mercury transferred from quadrant A to quadrant B in altering the

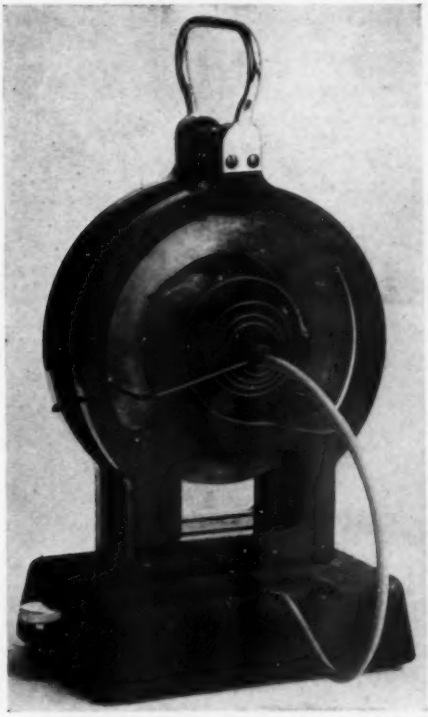


FIG. 16.

relative level of the surfaces. The steel disk therefore revolves until the spring is wound up to compensate for the change in balance. The angle turned through for small pressure differences is nearly proportional to the pressure.

A gauge constructed on this principle is shown in Figs. 15, 16, and 17. The steel disk is 15 cm in diameter and has a mercury annulus of

4 sq. cm cross section. The glass is secured with hard deKotinski cement. Lubrication is by butyl phthalate because the glycol compound softens the cement. The gauge is sensitive to a twentieth of a millimeter when the range is 25 mm for a swing of 150 degrees.

The sensitiveness is controlled by the strength of the spring and can be made so great that the dial swings through a right angle for a couple of millimeters' pressure-change. The accuracy is then very

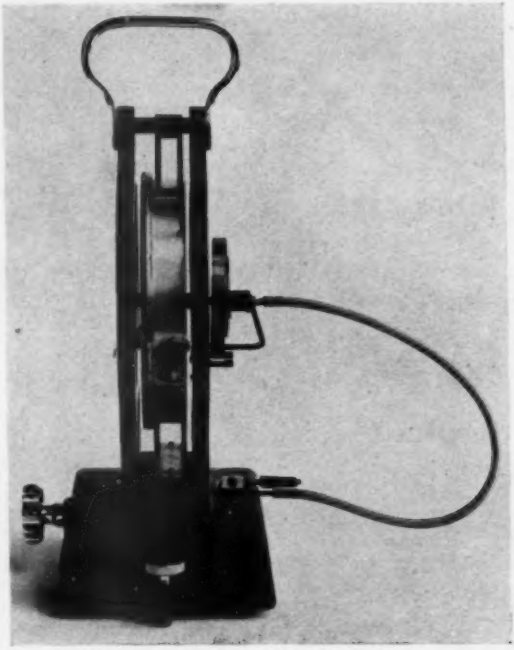


FIG. 17.

poor and the hysteresis large, the dial refusing to return to any very definite zero. This is due partly to the difficulty of boiling out the dissolved air, partly to friction in the ball bearings and rubber lead-in tube. The dial gauge is no substitute for the lever pattern and is intended for an entirely different field. It is useful on the lecture table, in the biological laboratory, and in the power house for reading steam condenser pressures.

Figures showing the performance of a gauge with stiff spring are given in Table 2. The hysteresis is 2 mm of mercury for a complete double swing from atmospheric pressure to vacuum and back. When held in constant use so that it is matured at low pressure, the lag for

TABLE 2.

Pressure in mm mercury	Deflection in degrees	Increment
0	0	
1	5	5
2	10	5
3	15	5
4	20	5
5	25	5
6	30	5
7	35	5
8	40	5
9	45	5
10	50	5
11	55.2	5.2
12	60.4	5.2
13	65.8	5.4
14	71.3	5.5
15	77.0	5.7
16	82.4	5.4
17	88.0	5.6
18	93.4	5.4
19	98.7	5.3
20	104.1	5.4
21	109.6	5.5
22	115.0	5.4
23	120.5	5.5
24	126.0	5.5
25	131.5	5.5
26	137.0	5.5

a 5 mm deflection around a mean pressure is about one-tenth of a millimeter. The response is practically instantaneous to changes of pressure.

OTHER MODIFICATIONS

Pendulum Gauges.—The principles underlying the two gauges may be applied in many ways. The compensation for the gravitational transfer of mercury may be provided by a pendulum fastened to a symmetrical square U-tube at its swinging center. This yields the

type at the right of Fig. 18. Since, however, a very long heavy pendulum is required for a small U-tube, it is better to let the glass work and mercury contents take the place of the plumb bob, as suggested in the left of the figure. An apparatus of this design was constructed to record on a flat chart rotated on one of the well known commerical clock drums. The deflection-pressure curve is nearly linear and the accuracy is very high.

Levelling Instrument.—The lever gauge gives its magnified reading because each transfer of mercury caused by change of pressure induces a large extra gravitational transfer, compensated by the dash pot

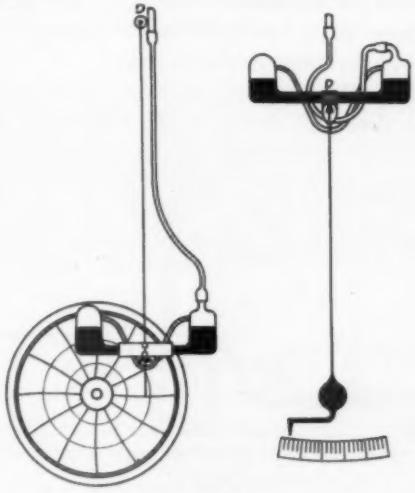


FIG. 18.

mechanism. A change of level of the fulcrum relative to the dash pot produces an equally magnified effect. A gauge reading one centimeter deflection of pointer for a millimeter pressure change will read a centimeter for a millimeter level change. The gauge therefore constitutes an accurate levelling instrument when both ends of the U-system are left open to the atmosphere. Conversely in normal use, the gauges must be carefully levelled and held on a base showing no seasonal variation. A wooden bench supported on wooden legs has been found quite unsatisfactory for long continued readings, and recourse was made to a stone slab fastened by iron angle pieces to a stone wall.

The gauge can be made substantially insensitive to changes of level by inclusion of a second dash pot at the other end of the base plate. The pots must communicate with one another below the surface of the mercury, and they must be so arranged in relative size and position that tilting the base plate raises the level of the fulcrum and the mercury in the plunger dash pot by the same amount.

The dial gauge is almost insensitive to level changes.

A Recording Barometer.—If both ends of the U-tube system, *A*, *B*, *F*, of Fig. 5, are kept open, an alteration of buoyancy anywhere will cause the cradle to swing. Pouring more mercury into the dash pot has as much action for millimeter change of level as driving mercury from *A* to *B*. The property can be utilized to construct a self-recording addition to the standard mercurial barometer.

Referring to Fig. 19, a balance cradle *A*, *B*, *F* is mounted with *F* dipping into the receiving cup of a wide bore mercury barometer.

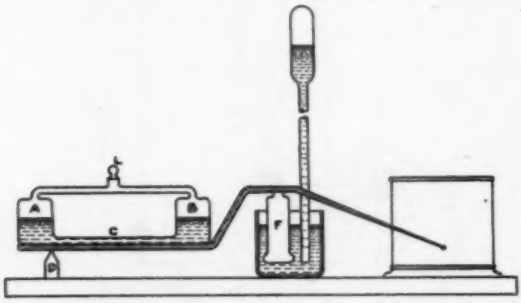


FIG. 19.

Variations in barometric height will make diminished variation in the level in the cup, but these will be magnified by the movement of the plunger, so that any desired amplitude of movement may be recorded on a clock driven chart.[21]

An instrument has been in use for some time in this laboratory and has been found to possess several valuable qualities. Since the plunger is withdrawn when the mercury in the dash pot rises, the variation in cup level at the base of the barometer is less than normal, necessitating less adjustment of the barometer scale. The presence of a standard barometer as an integral part of the recording instrument

[21] The readings are, of course, inverted, a high barometer giving a low swinging cradle. Correction can be made by securing the pointer to the other end.

enables a check to be made whenever the record is in doubt. The records given are much freer from hysteresis than those from the aneroid type, and the ability at will to secure a two to one or ten to one magnification of reading fits it especially for research work. In Fig. 20 are shown two barometric traces secured for the week December 28, 1928—January 3, 1929 in Rochester, N. Y. The traces were obtained separately and have been superimposed photographically. The line of less amplitude was given by a barograph of well known make while the other came from the pen of an adapted vacuum gauge working at a 3 1/2:1 magnification. The improved rendering of the latter is particularly marked at the points *A* and *B* where certain slight fluctuations are absent from the barograph curve.

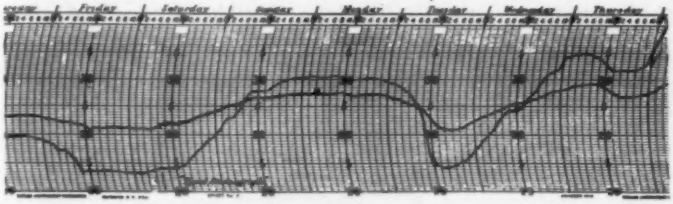


FIG. 20.

For use as a barograph it is important that all the mercury surfaces should be lubricated. The barometer should have a couple of drops of butyl phthalate[22] on the surface of the torricellian space; and there should be about a cubic centimeter in the dash pot. The limbs *A* and *B* in the swinging cradle may be covered with a little 1% aqueous mercurous nitrate containing sufficient nitric acid to give a clear solution, and should be protected from evaporation as shown in Fig. 15.

SUMMARY

A need has been demonstrated for a vacuum gauge recording between the limits 5 mm and 0.01 mm.

A differential mercury manometer has been described which covers the range satisfactorily.

The principle has been extended to construct a moving dial instrument.

The gauges are shown to yield a nearly linear deflection with pres-

[22] This will be found a very valuable addition to all barometers.

sure and their use is extended to obtain absolute measurements of pressure.

Certain organic liquids are recommended for lubricating the mercury meniscus.

A number of other devices are described.

APPENDIX

A Vacuum Check Valve.—Releasing the vacuum suddenly is injurious to the gauges described above. The lever gauge requires re-evacuating and the dial pattern is smashed to pieces. It is convenient therefore to include a valve in the pipe line which will close if the pressure is accidentally increased.

The common check valve wherein a ball, plate, or plunger is driven by the incoming gas against an orifice, is well known. It has this drawback for vacuum use that, if actuated positively by a spring, it offers too much resistance to the gas flow. If operated by the gas itself, a harmful influx may take place long before the ports have closed.

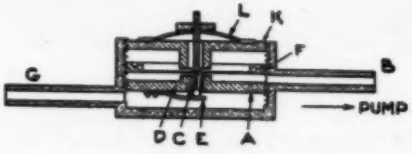


FIG. 21.

A check valve was therefore designed, following the principles of the Beard gas regulator, to operate positively by atmospheric differences leaving the vacuum path entirely unmolested when in the working region.

Fig. 21 shows an elevation of the valve. A brass cup two inches in diameter and one inch deep is closed at the bottom and threaded in the cylindrical portion. Two side tubes *G* and *P* make entrance. Screwed in first is a keyed disk *A* having a conical recess *C*, communicating with a central hole and collar *D*. In the recess a steel ball (from a ball bearing) is secured by a leaf spring *E*. On top of this a thin corrugated steel diaphragm soldered into the brass ring *F* is screwed home. The diaphragm is provided with a plunger taking the form, on one side, of a steel pin, and on the other, of a finely threaded screw. A third disk *K* with collar and a stout leaf spring *L* and a milled nut completes the arrangement.

During assembly the mouth is placed at G while F, the diaphragm, is screwed down until air just passes by the ball. The plate K is then screwed in and the spring tension adjusted by the nut M until the plunger is just withdrawn from the ball, thus closing the air passage. Adjusted so the diaphragm should be resting firmly against the collar K, P is now connected to the pump and G to the gauge. M is screwed home as far as it will go so that there shall be no passage between P and G. When the pump has reduced the pressure to the working region (say 2 cm of mercury), M is unscrewed until atmospheric pressure outside the diaphragm just overcomes the spring tension and moves the plunger against the ball. There is now a clear path between G and P. If, for any reason, the pressure is raised, either gently or suddenly, on the pump side, the diaphragm is withdrawn and the ball returns to its seating. The pressures involved are great and the inertia of the parts is small so that response is virtually instantaneous. The device has been tested by placing between a common U-tube manometer and oil pump taking care that the volume of the connecting tube shall be as small as possible to give a sudden leak the greatest chance of doing damage. On pulling off the connection to the pump the mercury rose from zero to three centimeters and then took thirty seconds to rise the remaining centimeters to the top of the column. The work of actuating the valve is thus performed by the pump on the one hand and the outside atmosphere on the other. The rarefied gas stream has no work to do. The valve can be recommended confidently for including any vacuum line not reaching below one-thousandth of a millimeter of mercury. The screw rings in this case are sealed in position with thick shellac cement and baked out. For very low pressures solder may replace the shellac. When there is any water vapor or acid in the exhausted gas, the steel ball and spring should be replaced by ones of phosphor bronze.

EASTMAN KODAK COMPANY,
ROCHESTER, N. Y.
DECEMBER 1, 1928.

Lichtelektrische Erscheinungen. B. Gudden. J. Springer, Berlin. 325 pages. (Struktur der Materie VIII.)

While a great number of investigations have been devoted to the subject of emission of electricity by radiation only very few of the problems have been solved. This volume presents the main experimental results without going too much into theoretical speculations. The first nine chapters are

devoted to a description of photoelectric emission from the surface of solids and liquids and constitute approximately one half the book. The photoelectric conductivity of crystals, on which much light was recently thrown by the experiments of Gudden and Pohl, is then taken up in considerable detail. This is probably the most valuable part of the book. A separate chapter deals with the photoelectric effect in gases, which, theoretically, is much clearer but only recently has been successfully investigated experimentally. The effects of x-rays and γ-rays are considered very briefly without any attempts to make the presentation complete. A bibliography of the photoelectric effect from 1914 up to the end of 1927 contains titles of more than 600 papers.

As a whole the book is clearly written and well illustrated with diagrams and figures.

A. WOLF

Empfindliche Galvanometer. Für Gleich- und Wechselstrom. By Dr. Otto Werner. viii+208 pp. Walter de Gruyter and Company, Berlin (W 10, Genthiner Strasse 38). Paper, Rm 13; bound, Rm 14.

It is the purpose of this volume to present concise information regarding the theory, construction, and use of the various types of galvanometers and accessory devices employed by engineers, physicists, physiologists, teachers and all others having occasion to measure small electric currents, alternating or direct. It is the purpose of the author to assist the worker in selecting a measuring instrument best suited for the problem at hand.

The major portion of the book—some 160 pages—is divided into two sections: Galvanometers for direct current, and galvanometers for alternating current, respectively. In the former are three subsections on Moving-coil Galvanometers, Moving-magnet Galvanometers, and String Galvanometers. The latter contains four sub-sections, viz: Vibration galvanometers, Electro-dynamometers, Electro-thermal Devices and Oscillographs. Special attention is given to questions of sensitivity and limits of sensitivity. Numerous tables are given showing the comparative characteristics of various makes of galvanometers of a given type. The book concludes with three short sections on such topics as indirect methods for measuring small currents (electrometers, amplifying devices, etc), optical accessories for reading deflections, and the choice of a proper galvanometer. There is also included a classified list of the more important literature references.

Space limitations have probably necessitated the omission of much material which one would expect to find in such a book. For example, the reference to electrometers is disappointingly short; there is no mention of methods of measuring ionization currents; there is only a brief discussion of amplifying devices. Nevertheless, we remember that the title of the book is Sensitive *Galvanometers*, and as such it makes a useful addition to the literature of the subject.

F. K. RICHTMYER

A MODIFIED PIRANI VACUUM GAUGE

By Thos. De Vries

A modified Pirani vacuum gauge can be made by assembling four carbon or tungsten bulbs in a Wheatstone bridge arrangement, and connecting to the vacuum line those which are in the opposite arms of the bridge. The other two bulbs serve as balancing resistances, together with a dial-decade resistance box in series with one of these bulbs to balance the bridge. The value of this resistance is a measure of the pressure. This gauge, like the single bulb type of Pirani, can

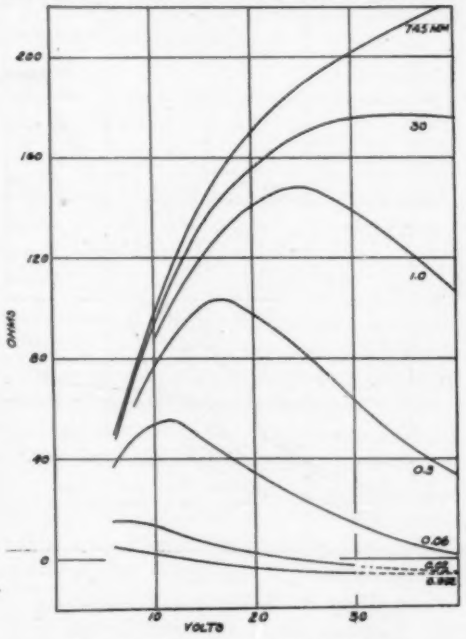


Fig. 1. *Balancing resistances for carbon bulb Pirani guage.*

be used either at constant current or constant voltage. Instead of the resistance in the decade box, the voltage across the galvanometer connection can also be used as a measure of the pressure in the gauge.

The above arrangement becomes a simple and handy set-up for controlling low pressures or detecting slight changes. Such an arrange-

ment is used by Mr. R. H. George to control the pressure in a cathode ray oscillograph of his design. The author has therefore investigated what the optimum conditions of sensitivity are, when the ordinary type of incandescent lamp bulb is used.

Fifty-watt carbon bulbs were found to be very desirable, more so than 75 watt tungsten bulbs of the gas-filled variety, evacuated before being used. Great care had to be taken not to oxidize the tungsten wire; nor was the change of resistance with pressure as reproducible and pronounced as with the carbon bulbs.

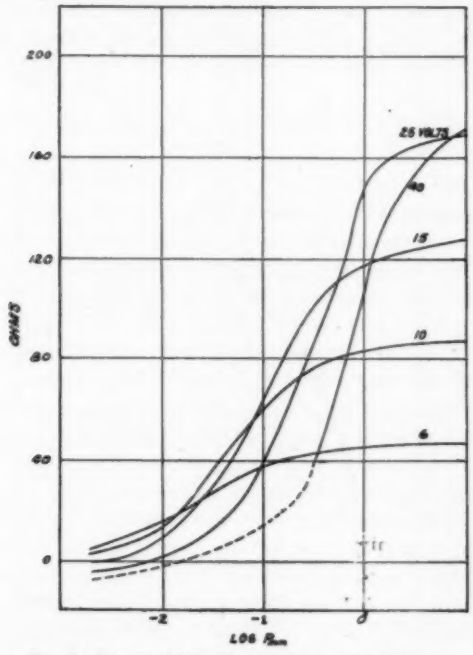


FIG. 2. *Change of balancing resistance with pressure.*

The results show that there is an optimum voltage (or current) at which the largest change of resistance (or voltage drop across the galvanometer) with pressure is obtained. Figures 1 and 2 show the values of "Balancing Resistance" for different pressures, and voltages. The change of "Potential" with "Pressure" at constant voltage or even constant current belong to the same family of curves and therefor are not reproduced here. If the gauge is used with optimum conditions,

a fluctuation in the current will not influence the balancing resistance very much, but a slight pressure change will. Thus with 25 volts across the Wheatstone bridge, a change of pressure from 1.0 to 0.1 mm meant a change in the balancing resistance to the extent of 110 ohms. An unbalancing of one ohm deflected the needle of a Leeds and Northrup student galvanometer four divisions. This pressure change can therefore be detected to one part in 4,000. On the other hand 15 volts is better to cover 0.1 to 0.01 mm with a resistance change of 57 ohms or one part in 140. The lower limit of detectable pressure was in the neighborhood of 0.001 mm.

(It was interesting to note that the adiabatic cooling of air under a pressure of only 0.01 mm would unbalance the bridge sufficiently to throw the galvanometer off the scale.)

CHEMICAL LABORATORY,
PURDUE UNIVERSITY,
LAFAYETTE, INDIANA.

Ophthalmic Lenses. By H. H. Emsley, Head of Applied Optics Department, Northampton Polytechnic Institute, London, and Wm. Swaine, Lecturer in Physics, West Ham Municipal College. vii+319 pp. The Hatton Press, Ltd., 72–78 Fleet St., London. 15/, postage extra.

The authors have prepared this book, based on lectures given at the Northampton Polytechnic Institute and the West Ham Municipal College, to "fill one of the several gaps existing in the literature needed by students training to become opthalmic or dispensing opticians."

The first eleven chapters present the everyday practice of testing and marking thin spectacle lenses and prisms. The treatment is elementary and requires only a minimum of previous knowledge and experience. The remaining chapters, XII to XVII inclusive, deal with the subjects of lens form and thickness, image formation by astigmatic lenses, aberrations of lenses, spectacle lens design, etc. Here an elementary knowledge of geometrical and visual optics is required.

A very valuable feature of the book is the carefully prepared list of exercises and questions at the end of each chapter to assist the student in acquiring a working knowledge of the principles presented.

While the book is primarily intended for the training of professional opticians, it will be found useful as a reference book for students of general physics who are interested in getting a glimpse of one branch of applied optics. Teachers of physics will find the problems particularly helpful.

F. K. RICHTMYER

Amateur Telescope Making. Edited by Albert G. Ingalls, Associate Editor, Scientific American, with contributions by Russell W. Porter, Charles S. Hastings, William F. A. Ellison, George Ellery Hale, Elihu Thomson, and others. xii+285 pp. The Scientific American Publishing Company, New York. $3.00, prepaid. (Foreign, $3.25).

The most satisfying thrills which any of us experience come from having accomplished something by our own efforts. We may scale a mountain, direct a great enterprise, unravel the secret of nature, or produce bumper crops from the soil. Be it vocation or avocation, professional or amateur, the thrill has a common origin. Those who find satisfaction in having created something with their own hands will find AMATEUR TELESCOPE MAKING a most interesting, and valuable book. But the satisfaction need not stop with mere creating. For the amateur astronomer can use his home-made telescope "advantageously for science" by collaborating with the professional astronomer in making important observations within reach of "a telescope of three inches aperture or larger."

Professionals and experienced amateurs have united in preparing this book to place in the hands of the uninitiated the technique of making and mounting telescopes. An inexpensive outfit of tools, costing "no more than a very few shillings," a pair of equal glass disks, a barrel or "better still two barrels," a pound or two of pitch, carborundum, rouge ("jewelers"!), a little spare time and a reasonable amount of patience and enthusiasm—these are the essentials. Skill, of course! But that is gradually acquired by carrying out the various processes so fascinatingly described in the various sections of the book.

And then: "No chapters in the history of science are more inspiring than those that recount the discoveries of the amateur." (Quotation from the section by G. E. Hale). To make a telescope with one's own hands, and then to use it for scientific purposes! What could be more thrilling!

F. K. RICHTMYER

ON THE ULTRAMICROMETER OF DOWLING

By Stig Ekelöf

The article by Mr. Obata concerning the ultramicrometer, published in June (1928) number of this Journal, has given rise to the following pages on the same subject. The purpose is to add to the communications of Mr. Obata and other authors some, as I hope, new results, obtained in our laboratory, where the author, under the guidance of Prof. W. Weibull, has been occupied with the Dowling circuit during the last year.

1. GENERAL ARRANGEMENT. SENSITIVITY

(a) The only method used by us is the "*capacity-change method*". Fig. 1 shows the circuit first tried, the usual tuned anode circuit with separate inductive reaction. While examining this circuit we soon found that an appropriate grid battery increased the sensitivity appreciably. This fact has also been confirmed by a theoretical calcu-

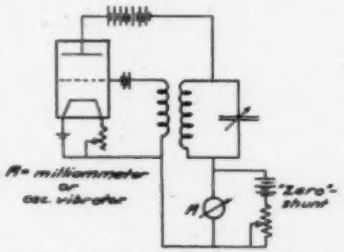


Fig. 1. *"Ultramicrometer circuit" first used by the author. The grid battery increases the sensitivity.*

lation. In order to get high sensitivity, the grid voltage must be such as to make the tube work around a point in the lower curvature of its anode-current vs grid-voltage characteristic. The exact value of the grid bias is best found experimentally. In this manner we reached a sensitivity of 0.1 milliampere per 1$\mu\mu$F variation of capacity. (Tube: Telefunken RS 228).

The records were made by a Siemens oscillograph. The vibrator current amounted to 4 mA at maximum deflection. In spite of this small value it would often have been necessary to apply an amplifier between the ultramicrometer circuit and the oscillograph. However

it was found that it was possible to further increase the sensitivity of the ultramicrometer circuit.

Using the little amateur transmitting tube just mentioned in the circuit Fig. 2, we achieved the astonishing result of *an anode-current variation of 4 milliampere per 1μμF capacity-variation*. The principal novelty shown by Fig. 2 is, that the grid-battery has been replaced by a *gridleak condenser combination*. An accurate study shows furthermore, that the arrangement is influenced in the least possible way by stray capacities, such as may be introduced by the body of the experimenter. On this account it is not necessary to shield the entire apparatus. It is sufficient to place the condensers in shielded and grounded boxes.

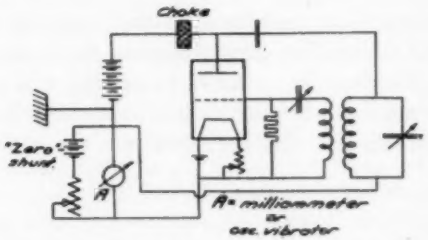


FIG. 2. *A circuit with grid leak and grid condenser, giving a sensitivity of 4 mA per μμF capacity variation.*

The choice of a suitable tube seems to be of the greatest importance for the good functioning of the gridleak circuit. Different tubes have been tried. Most of them however gave unsteady curves with sudden jumps. The tube giving the sensitivity mentioned has the following data: "Telefunken RS 228; 5 w. amateur transmitting valve."

Filament current $I_f = 1.1$ amps.
Plate voltage $E_a = 220$ volts
Saturation current $I_s = 200$ milliamps.
Internal resistance $R_i = 3500$ ohms
Amplification factor $\mu = 6.5$

The values of gridleak and condenser should be determined empirically. A small gridleak decreases the sensitivity; too large a leak gives unstable curves. Five thousand ohms is a proper value. The grid condenser should be chosen rather small, usually $< 100\mu\mu F$. Small capacity variations of this condenser will also influence the anode current, so that

it is possible to obtain a high sensitivity by measuring these variations. We have never tried this mode however.

As to other data, the following may be mentioned: The frequency of the tuned circuit is in the order of 500 kilocycles, the coils are usual honeycomb coils.

(b) One can easily calculate, that with a sensitivity of 4 mA per $\mu\mu$F and the aid of a good galvanometer, it would be possible to measure without difficulty a displacement of 10^{-9} mm, i.e. about 1/100 of the diameter of the hydrogen atom! Now, naturally, there is no sense in speaking of such small mechanical displacements. We must also fully agree with Mr. Obata in pointing out that the difficulty of maintaining the anode current constant for a longer time limits the possibility of such extreme measurements.

This is especially the case with the tube used by us, because its filament current is very small in comparison with the saturation current. Extremely small variations of the filament current I_f, therefore, have a considerable influence on the anode current I_a. The author has found, that if an instrument of max. 0.1 mA is used for measuring the variations of I_a, the I_f should not vary more than 1/50000. It is hardly possible to reach such extreme constancy, but with the aid of large-capacity batteries and taking other precautions it is not difficult to use an instrument of max. 0.5 mA. With two condenser plates 40 mm in radius, placed 0.1 mm from one another, this would allow us to measure displacements less than 10^{-6} mm.

We have however not yet made use of such extreme measurements. The advantage of the ultramicrometer in engineering lies more in its applications to measurements of minute vibrations than of small finite displacements.

2. APPLICATIONS

Concerning applications, we have used the Dowling circuit for the examination of small, rapidly varying pressures and mechanical vibrations of different kinds. By recording, for instance, the vibrations of a motor shaft initiated by the blow of a hammer, one has a simple means of determining the critical speed with regard to bending vibrations.

In the following we shall describe an application of special interest, because it was made in order to solve a problem met with in engineering practice. The investigation was performed in one of the hydroelectric plants of the Swedish Government. The problem was as follows:

Before the water reaches the turbines, it passes racks, which protect

the water wheels from ice. The design of the racks concerned is shown in Fig. 3. They consist essentially of long steel bars or lamels A (cross-section 15×220 mm), onto which are welded transversal bars C. It soon appeared that the lamels broke as shown by Fig. 3; e. g. at the places where they are united with the transversal bars. These cracks were explained as coming from fatigue of the material, caused by vibrations of the lamels. In order to confirm this hypothesis it was necessary to measure the vibrations and this was made with the ultramicrometer. The difficulty of making such measurements at a depth of 6 m below the surface of water was that no fixed point was

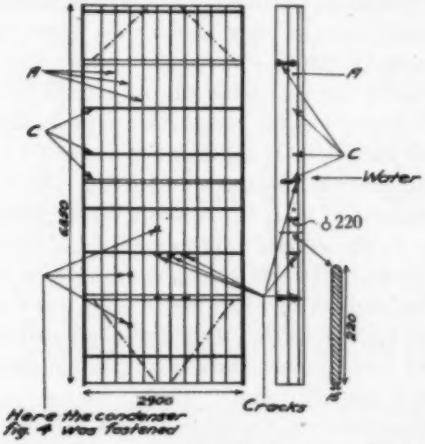


FIG. 3. *Racks upon which vibration tests were made.*

available in the vicinity of the lamels. Where could we obtain a fixed point? The following solution was found:

The condenser, the capacity variations of which indicated the vibrations of the lamels, was designed according to Fig. 4. A is a brass box, on the bottom of which is placed an insulating disk B. On B is assembled the brass disk C, constituting the one plate of the condenser. C is electrically connected to the other parts of the measuring device by means of the cable D. The second plate is formed by the disk E, supported by the spring F (C and E are 50 mm in diameter). The air gap between C and E can be regulated by the screws G. The box is made watertight by the cover H.

Now let the box be fastened to the point of the lamel, where the vibrations are to be measured. We suppose that the free oscillations

of the disk E and the spring F have a frequency much lower (say $5\times$) than that of the lamel vibrations. Under such circumstances E will not be able to follow the vibrations of the lamel and the other parts of the box; the air gap between C and E will vary according to the lamel vibrations. In the movable part E we have found our fixed point!

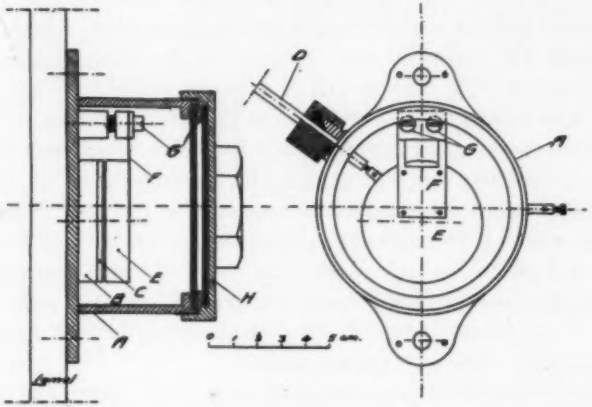


FIG. 4. *Condenser, used for measuring the vibrations of the racks, Fig. 3.*

Because of the large capacities introduced by the cables connecting the box with the other parts of the apparatus, it was not possible to reach any high degree of sensitivity (only $75\mu\mu$F per mA, which however proved quite sufficient). Fig. 5 shows some of the records obtained.

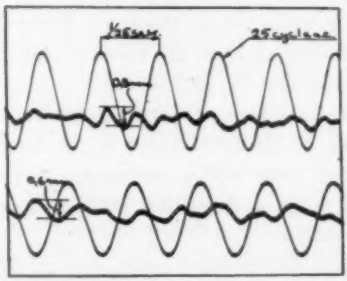


FIG. 5. *Records of the vibrations of a rack Fig. 3, obtained with a Siemens oscillograph. The measurements are made at a depth of 6 m below the surface of water.*

We used a Siemens oscillograph with a vibrator having a sensitivity of about 13 mm deflection per mA.

ROYAL INSTITUTE OF TECHNOLOGY,
STOCKHOLM, SWEDEN.

A CIRCUIT-BREAKER FOR WATER-COOLED X-RAY TUBES

By L. W. McKeehan and L. Merle Kirkpatrick*

The usual type of circuit-breaker for the protection of water-cooled x-ray tubes acts when the water pressure at the intake passes certain limiting values. The rise or fall of pressure which trips the circuit-breaker may be due to an obstruction in the cooling system, to leakage, to failure of the pumping system or to other causes which may effect the efficiency of cooling, and which should be investigated by the operator.

Such a circuit-breaker appeared unsuited to the conditions in our x-ray laboratory. This laboratory is at the top of the building, which stands on a hill. The pipes supplying it are of small diameter. The water pressure is accordingly low and subject to relatively large fluctuations. A pressure-controled circuit-breaker with reasonably safe operating limits would act far too often.

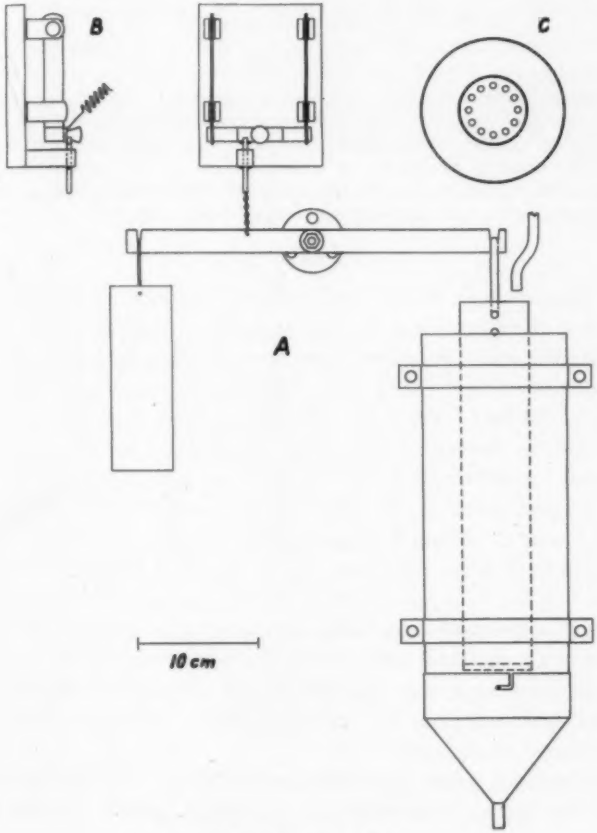
The device shown in front elevation at *A* in the figure was designed to meet this situation and does so. It consists of a bucket receiving the exit water from the cooling system and provided with an adjustable leak so that the desired rate of flow is just more than sufficient to keep it full. The excess spills out through four holes near the top, one of which is seen just below the bail, into a catch-basin which also receives the main flow. The bucket hangs on a lever with a counterpoise and this counterpoise pulls the switch-latch if the bucket becomes too light. The switch, shown again at *B*, is jerked open by a strong spring. With the dimensions shown the pull on latch can be made greater than 4 pounds, so that a good deal of friction can be tolerated. The adjustable leak consists of threaded holes in the bottom of the bucket, shown in section at *C*, into which screws are put as plugs. For one molybdenum target tube of the usual form two openings out of the twelve were found sufficient.

The lever stud and other fixed parts are mounted on a wooden backing directly over a sink. The switch in its open position make a contact (not shown) which sounds a call for the operator.

The volume of the bucket is such that a complete water stoppage causes the circuit-breaker to act in about 5 seconds. Less drastic

* Deceased. Late Sterling Fellow, Yale University.

changes can be endured for longer periods. An over-supply of water (hardly possible under our conditions) has no effect. A study of pos-



A. Front elevation of assembly.
B. Side elevation of switch.
C. Section through bucket and catch-basin.

sible casualties indicated that control by exit flow is inherently safer than control by intake pressure.

SLOANE PHYSICS LABORATORY,
YALE UNIVERSITY,
JANUARY 26, 1929.

TRIBOELECTRICITY OF QUARTZ AND MERCURY

By L. H. Dawson

ABSTRACT

The adsorbed water vapor upon the surfaces of fused quartz and mercury was shown to have a decided effect upon the triboelectric charge produced upon the surfaces. When the water vapor was expelled from the quartz surface by proper heating and exposed surface of the mercury cleaned by distillation, the triboelectric charge was reproducible and was 1 E.S.U/cm² within the limits of error of the experiment. The method of making contact was shown to have an effect upon the magnitude of the triboelectric charge.

INTRODUCTION

The phenomenon of the production of an electric charge by the separation of surfaces was known as early as the 6th century before the Christian era.[1] We have today very little insight into the causes of the phenomenon or the factors affecting the magnitude of the charge produced. Morris Owen,[2] by rubbing together specimen conductors and insulators, showed that up to a certain point the charge produced was nearly proportional to the work done; but above this point the charge reached a maximum value. Jones[3] extended Owen's work with the same results. Many lists giving the triboelectric relation between substances have been published, one of the most extensive is that due to Shaw.[4] Work of a quantitative nature was reported by Richards[5,6] in which he determined the charge produced upon surfaces of cylinders of conducting material and dielectrics when wrung together. For glass and steel he found that 21esu/cm² of positive electricity was produced upon the glass and an equal amount of negative electricity was produced on the steel.

In the present paper the triboelectric charge of fused quartz and mercury for dry air at atmospheric pressure is given. Moisture upon the surfaces is shown to have a marked effect upon the magnitude of the charge as does also the character of the making and breaking of the contract between the surfaces. The effects of separating the surfaces of quartz and mercury in gases at reduced pressures are discussed.

[1] A Treatise on Frictional Electricity. Sir. W. S. Harris.
[2] Phil. Mag., *17*, p. 457; 1909.
[3] Jones, Phil. Mag., *29*, p. 272; 1915.
[4] P. E. Shaw, Roy. Soc. Proc., *A94*, p. 161; 1917–1918.
[5] H. F. Richards, Phys. Rev., *14*, p. 290; 1920.
[6] H. F. Richards, Phys. Rev., *22*, p. 122; 1923.

APPARATUS

The main chamber of the apparatus consisted of a pyrex cylinder *A*, Fig. 1, 3 cms in diameter and about 30 cms in length supported in a vertical position and connected to a MacCleod gauge and diffusion pump. From the top of this cylinder, by means of glass tongs, was supported a circular disk of fused quartz (*B*), Fig. 1, about 12 mm in diameter and 5 mm in thickness with the bottom surface ground plane to within 1/4 wave length of light. A Faraday cylinder *C* in contact

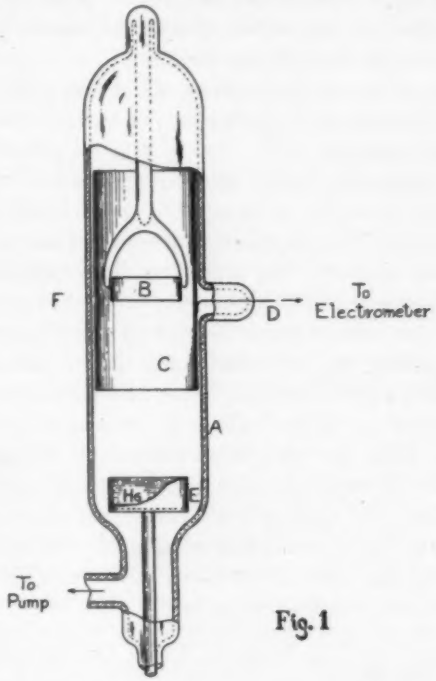


Fig. 1

with a lead to the outside surrounded the quartz. A steel cup of somewhat greater diameter than the quartz and capable of being raised and lowered from the outside of the vacuum by means of an earthed steel rod, contained pure mercury. The motion of this cup was limited by a micrometer head, and contact between the mercury and quartz could be observed through a peep hole *F*, Fig. 1 in the Faraday cylinder. Upon breaking the contact between the quartz and mercury, a charge was produced upon the quartz and of course an equal and opposite

charge was induced upon the Faraday cylinder. The magnitude of this charge was measured by a well shielded goldleaf electrometer[7] of small and constant capacity. Previous to each determination the apparatus was evacuated and the tube heated to 420°C to insure of clearing the surfaces of the adsorbed gases.

A glass reservoir placed in such a position that the mercury could run into the steel cup under the action of gravity, the surplus overflowing into a suitable basin, served to replenish the supply in the cup. A clean surface could thus be obtained quite readily. To further insure a clean surface the mercury in the cup could be boiled off to any desired amount by a suitably constructed furnace.

The capacity of the electrical system, which was small, was measured by comparing it with the capacity of a standard condenser by the heterodyne beat method.

Experiments performed in air at a pressure of 76.6 cms of mercury and at a relative humidity of about 40% showed that the magnitude of the charge increased as the number of contacts broken until a maximum charge was reached. The maximum charge appeared very variable from day to day, due in all probability to the change in the character of the surfaces in contact from the film of water vapor adhering to them. To eliminate this adsorbed water vapor, the apparatus was baked out at about 420°C for four hours under a pressure of less than .001 mm of mercury. After refilling the mercury cup and distilling off the surface, clean, dry air was allowed to fill the apparatus. The phenomenon of an increasing charge with increasing number of contacts until a maximum was reached was again observed; but without the presence of water vapor the maximum charge could be repeated from day to day within the limits of experimental error. The following table gives a sample of the results obtained:

Exp.	Mean charge per unit area ESU	Max. dev.	Max. % dev.	
1	.929	.923 − .934	.65%	Press. 76.6 cm
2	1.072	1.062 − 1.080	.93%	Time of contact 20 seconds.

While each separate contact at atmospheric pressure produced a constant deflection of the electrometer, at reduced pressures, namely

[7] J. C. Hubbard, Phys. Rev., *33*, p. 558; 1911.

66 cm, 2 cm, 10 mm, .01 mm, .001 mm of mercury, the deflection did not remain constant but fell from a maximum value to a lower value which became constant after an interval of time. This constant value appeared to be approximately the same for all pressures except at about 2 cms and the time taken to reach this value was about 60 seconds.

The electrometer indicated a positive charge upon the quartz when the mercury was taken away, thus there appeared to be a deficiency of electrons at the surface of the quartz, the mercury apparently tearing them from the surface. The potential gradient between the quartz and the mercury brought about by the breaking of the contact, was of a great enough magnitude to ionize the gas to some extent and consequently make it conducting. Electrons passed from the mercury to the quartz thus neutralizing some of the positive charges with a consequent collapse of the gold leaf. The passage of the electrons took place until the supply of ions was exhausted, when the deflection remained constant.

That this was an ionization current was further substantiated by the fact that at approximately 2 cms of mercury pressure the gold leaf, deflected to a large value immediately upon the separation of the surfaces and on a further separation to about one and one-half millimeters distance, the gold leaf fell back to a zero position. There was apparently an almost complete discharge of the quartz. Now it is well known that under given potential gradients there is a pressure at which ionization takes place most readily.[8] That the potential gradients produced were of an order high enough to produce ionization may be seen from the fact that when mercury and quartz were separated by about 1.5 mm after contact there was a difference of potential of about 350 volts. It was at this separation that the discharge occurs.

In all measurements the contacts were made as gently as possible, and under these conditions an accurately reproducible charge was obtained. When the contact was made violently the electrometer deflections were very erratic and very large. Likewise an abrupt breaking of the contact between the mercury and quartz produced variable charges. With a gentle contact, it was found that when the break occured in about 0.25 seconds that the charge produced was approximately 10 percent higher than with a break of 2 seconds. The charges in either case were reproducible. The total length of time that the quartz and mercury were in contact affected the magnitude

[8] Thomsen—Conduction of Electricity through Gases—2nd Ed. p. 444.

of the triboelectric charge. With a short contact the charge was relatively large; but decreased to about 0.7 of this value for a contact of 15 seconds and to 0.5 after 20 seconds of contact. For contacts longer than 20 seconds the charge appeared to be constant.

This work was done at New York University under the direction of Prof. J. C. Hubbard.

HEAT AND LIGHT DIVISION,
NAVAL RESEARCH LABORATORY,
DECEMBER 28, 1928, WASHINGTON, D. C.

Introduction to theoretical physics. By Leigh Page, Professor of Mathematical Physics in Yale University. X+587 pp. D. Van Nostrand Company, New York. $6.50.

"This book, based on a course of five lectures a week given by the author at Yale, is designed as a text for first-year graduate students and for college seniors specializing in physics. Its aim is to provide the student in an elementary way with a survey of the whole field of theoretical physics." (Extract from the preface.) A previous knowledge of calculus is assumed. Vector methods and notation are used, but a sufficient outline of vector analysis is given in the Introduction (34 pages).

The book is divided into five parts: Dynamics, Hydrodynamics, Thermodynamics, Electromagnetism, and Optics and Spectroscopy. The scope of each section is such as would naturally be expected from its title. For example, the section on Electromagnetism contains three chapters, viz: Electrostatics and Magnetostatics, Electric Currents and Electromagnetic Theory. The section on Optics and Spectroscopy is divided into three chapters: Geometrical Optics, Physical Optics and Origin of Spectra. "The recent atomic theories of Heisenberg and Dirac and of Schrödinger have been omitted as being placed beyond the scope of this book by their mathematical complexities."

Even a casual inspection shows that the author has exercised rare good judgment both in the choice of subject matter and in the method of presentation. While the book includes a surprisingly large amount of material, requiring in places brief and concise treatment yet the author's purpose of "clear and logical presentation" has been admirably achieved. Of course, the book will serve as an excellent text, particularly if the student solves the problems appended to most of the articles. But it will also be useful as a reference book to refresh one's memory on here and there a formula, concept, or method of derivation. Profesosr Page is to be congratulated on producing such a useful volume.

F. K. RICHTMYER

A MODIFICATION OF THE SPECTROPHOTOMETER FOR OBTAINING MEASUREMENTS ON THE REFLECTION OF LIGHT FROM LIVING MATERIALS

By Louis A. Brunsting and Charles Sheard

Spectrophotometry is, without question, the most fundamental method for the analysis of color. Through the use of data obtained by spectrophotometric methods it is possible to describe color quantitatively in terms of its three attributes, relative luminosity (brilliance), dominant wave length (hue) and purity (saturation). By reason of the exactness of colorimetric analysis as made possible through spectrophotometry, either by the methods of reflection or of transmission of light, we find many applications of it today in the industrial world, as in the grading of oils, analyses of flours, paints and dyes, as well as in academic laboratories for the quantitative analysis of various substances in solution, hydrogen-ion concentrations and the like. By following the methods outlined in the report of the Committee on Colorimetry of the Optical Society of America[1] it is possible to analyze spectrophotometric data into red, green and violet excitation color values, and thence into dominant wavelength, purity and relative luminosity values, all on the basis of noonday sun as a neutral standard. While such analyses of spectrophotometric data, obtained either by methods of the reflection or the transmission of light, are not always necessary or of value, there are instances in which they contribute information which could not be obtained otherwise. This is true in the case of the color of the skin, for example, in which it is desirable to evaluate the rôle played by pigment and by blood, respectively.

Using a direct reading type of instrument, the Keuffel[2] and Esser color analyzer, Sheard[3] and Sheard and Brown[4] were among the first to apply spectrophotometric methods to clinical medicine in a series of observations on the color of the human skin. Further investigations, which are as yet unpublished, were carried on in the hope of establish-

[1] Troland, L. T. Report of Committee on Colorimetry. J.O.S.A. & R.S.I., *6*, pp. 527–596; 1922.

[2] Keuffel, Carl. A direct reading spectrophotometer. J.O.S.A. & R.S.I., *11*, pp. 403–410; 1925.

[3] Sheard, Charles. Analysis of the color of the skin and its significance. Science, *64*, pp. 70–72; 1926.

[4] Sheard, Charles and G. E. Brown. The spectrophotometric analysis of the color of the skin. Arch. Int. Med., *38*, pp. 816–831; 1926.

ing certain fundamentals regarding the color of the skin and of separating and evaluating the various factors responsible for changes in the color of the skin in different parts of the body, both in health and in disease.

In order to obtain values of the reflection of light of various wave lengths from the skin, which is a semi-opaque material, it is necessary to bring the area which is to be tested in close contact with the spherical lamp house which contains the sources of illumination. One of the magnesium carbonate standards is replaced by the object or substance the spectral reflection of which is to be examined. Easily accessible portions of the body, such as the back of the hand or the fingers, can be adjusted readily to one of the openings in the lamp house, although we have been compelled frequently to wait until the lamp house had cooled sufficiently before placing any part of the body in contact with it. While the drawing of air through the lamp house, as provided in the Keuffel and Esser color analyzer, is an aid in keeping the specimen under examination as cool as possible, we find that this is inadequate in the color analysis of living materials, whether these materials be tissues of animals or of plants. Variations in the temperature of the normal skin produce changes in color which are recognizable by the eye and hence must be eliminated if accurate data are to be obtained for the evaluation of the factors affecting the color of the skin as it actually exists.

In order to obviate the uncontrollable condition of temperature at the site of the specimen under examination and the consequent changes in the color of the skin, we have devised an attachment consisting of a water-cooled cell which can be substituted for the mechanism holding the magnesium carbonate blocks ordinarily found in the color analyzer. In 1926 Sheard and Brown (4, 1. c.) briefly described a modified holder for the standard magnesium carbonate block and the specimen. This modification consisted of an air-cooled concavo-convex metallic screen with its concave side turned away from the lamp house proper. This arrangement proved satisfactory when the length of time involved in a spectrophotometric examination was short, when the number of consecutive specimens to be examined was very small and when the areas to be tested were easily accessible, as the fingers or hands. However, cooling the cell by means of circulating water at room temperature or thereabouts insures much more certainly that changes in color do not occur, due to the effects of heat at the surface of the specimen under examination. The water-cooled attachment also permits of accurate

spectrophotometric data from the surfaces of living leaves intact with the parent plants, and similar materials. Furthermore, it is possible to study the effects of temperature on the color of the skin of a given part of the body, all other factors of environment and position of object under test remaining constant, through the control of the temperature of the circulating medium in the cell.

Fig. 1 is a photograph of the direct reading spectrophotometer with the modification consisting of the holder for specimen and standard magnesium carbonate block attached to the instrument. The dimensions of a convenient size of water-cooled cell, made of brass, are 15 cm wide, 20 cm long and 1 cm thick.

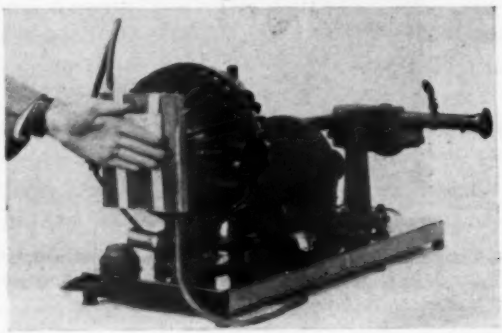


FIG. 1. *Keuffel and Esser color analyzer with water cell modification attached, showing relation of sample to be studied to the instrument.*

Fig. 2 shows a standard block (at A) coated with magnesium carbonate or with the same sort of material as is used to repaint the inside of the spherical lamp house and furnished by the manufacturers of the instrument. The block can be inserted in the aperture *B* of the water-cooled holder if curves for spectral transmission of solutions are desired. If reflection curves are desired the test specimen is brought to aperture *B* as shown in Fig. 1. The standard block is shown at *C*. This block can be attached to the brass plate of the water-cooled cell with shellac, de Khotinsky cement or adhesive tape. The standard plate is made of thin brass about 3 cm long and 2 cm wide. A second plate, about 3 mm thick, 2.5 cm long and 1.5 cm wide, is attached to this backing and after being coated with magnesium carbonate is ready for insertion in aperture *B* or *C*. Each of these apertures has a length of 2.5 cm and a width of 1.5 cm. Fig. 3 is a photograph of the water-cooled

cell taken from the side which is attached to the color analyzer and shows the character of the apertures, the white standards in position, brass strips and screws for setting the holder in position.

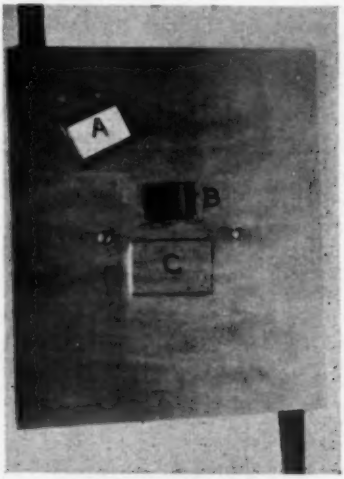


FIG. 2. *Water cell attachment to replace usual holders for standard and substance to be examined by reflection methods. A, brass plaque coated with magnesium carbonate; B, aperture in the water-cooled holder which is attached to the instrument; C, standard in position.*

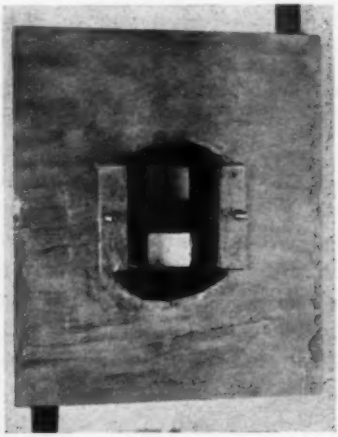


FIG. 3. *Photograph of the water cell with two standard magnesium carbonate blocks in position. View from side nearest to the lamp house.*

The sample of living material to be tested is brought flush against the upper opening of the water-cooled cell and is separated ordinarily from the metal of the cell by a thin layer of felt. In this manner it is possible to prevent any changes in the color of the skin which might be due to contact with the metallic wall of the water-cooled cell. The use of such a pad also tends to eliminate undue pressure of the part under examination, for such pressure would cause blanching of the skin due to obliteration of the blood capillaries at the surface.

There are two particulars in which this type of holder for the specimen and the standard white surface may be criticized. In the first place, surfaces used as standards must be kept standard. The slight inaccessibility of the plaque carrying the white standard in our device and the occasional difficulty in preparing white surfaces which are sufficiently smooth are not noticeable sources of inconvenience or inefficiency of operation. The standards can be checked against each other or a freshly prepared and smooth surface of magnesium carbonate (which can then be held in place before aperture *B*, Fig. 2) and by running rapidly through the spectrum one can determine whether or not the surface of the standard (*C*, Fig. 2) needs replenishing. In the second place, by reason of the depth of the apertures and therefore the greater remoteness of the specimen and standard from the sources of light as compared with the attachment on the original instrument, there is a loss of light reaching the spectrometer and hence some possible loss of accuracy in photometric determinations, especially at the two ends of the visible spectrum or possible inability to make determinations in the extreme red and blue regions. The difficulties which we have encountered in these respects, however, have been minor and are much more than offset by the ability to control the temperature and to eliminate other factors, such as pressure, which would introduce sources of error.

We have used this modification of the spectrophotometer in a number of problems of a biological character and judge that it may be of value to others of like interests or to those who have occasion to eliminate heating of specimens under test.

SECTION ON DERMATOLOGY AND
SECTION ON PHYSICS AND BIOPHYSICAL RESEARCH,
THE MAYO CLINIC AND THE MAYO FOUNDATION,
ROCHESTER, MINNESOTA.

A NULL METHOD FOR MEASURING THE STRENGTH OF A MAGNETIC FIELD

By David R. Inglis

There are a great many ways in which the strength of a magnetic field may be measured. The bismuth spiral,[1] the optical arrangements devised by Faraday[2] and Kerr,[3] the hydrostatic balance of Quincke[4] and various ballistic devices are among the most widely used methods. If one includes the various forms of fluxmeters, it may be said that the ballistic methods are the most used of all.

It seems to be pretty well understood that wherever a null method can be employed it is much more convenient and reliable than the readings of a ballistic galvanometer. One null method[5] has been suggested, and seems very useful for exploring permanent fields, but it requires the construction of a special commutating device. The following is a simpler method, and requires only such apparatus as is easily obtained in the laboratory.

DESCRIPTION OF APPARATUS

This method, like many others, is essentially a comparison of the field of an unknown coil with the field in a standard coil. The circuit diagram is shown in Fig. 1. X represents the coil of which the field strength per unit current is unknown. S is the standard solenoid whose field may be computed from its dimensions. The exploring coil E consists of many turns of fine wire. It should be placed at the center of the coil S and in direct connection with a galvanometer. In series with each of the coils, X and S, is an ammeter and a resistance. At least one of the resistances should be continuously variable. An emf is impressed simultaneously on both of these circuits by the same battery and key. The coils are so connected that their fields oppose one another at E. Before adjustment there will be a deflection of the galvanometer when the key is depressed or released. The variable resistance R_2 is adjusted for zero deflection of the galvanometer, under

[1] Lenard and Howard, Electrotech. ZS., *9*, p. 340; 1888.

[2] Verdet, Ann. Chim. Phys., *41*, p. 370, 1854; *43*, p. 37, 1854; *44*, p. 1209, 1859; *52*, p. 129, 1858.

[3] Kerr, Phil. Mag., *3*, p. 321, 1877; *5*, p. 161, 1878.

[4] Quincke, Wied. Ann., *24*, p. 374; 1885.

[5] Lane, Jour. Sci. Instruments, V, *7*, p. 214; 1928.

which condition the opposing fields at E are equal. Designating the field strength per unit current in S as H_s and that in X as H_x, and designating the current through S as I_s and that through X as I_x there exists the relation,

$$I_s H_s = I_x H_x$$

or

$$H_x = H_s \frac{I_s}{I_x} \tag{1}$$

I_s and I_x are read from the ammeters in the circuits and H_x calculated from (1).

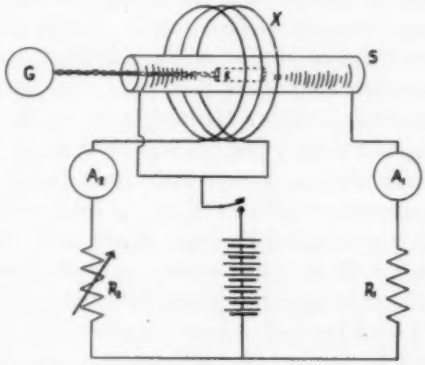


FIG. 1. *Null method for measuring the strength of a magnetic field.*

The type of apparatus and dimensions of coils used in any particular case may be determined largely by convenience. It may be that no long, evenly wound solenoid, with small enough diameter to fit into the unknown field, will be found in the laboratory. One may be easily wound with a lathe. In this study of magnetic field testing a coil of 1000 turns was wound on a glass tube, 23 cm long and 7 mm in diameter. At the center of such a solenoid the field per ampere is $4\pi n/10$ where n is the number of turns per centimeter. The end correction amounts to less than 0.05%, which is negligible for most magnetic work. The exploring coil, which was used, had 1000 turns of very fine wire in four layers about 2 cm long, and fitted snugly inside the standard solenoid. Its resistance was considerably less than that of the galvanometer. Carbon or water rheostats may be used in series with the coils S and X. The currents were in the order of an ampere,—too large for the use of ordinary resistance boxes. Before the measure-

ments are taken, the ammeters should be connected in series and checked with one another. One instrument may read a fraction of a percent higher than the other, and this fact should be taken into account in calculating the ratio of the currents in (1).

In the above discussion it has been assumed that the current reaches its final value within so short a period that there is no time for a deflection of the galvanometer to be caused by the slight inequalities of current during the period of growth. If this assumption is not satisfied, the galvanometer will be momentarily deflected. If this effect should be large enough to cause uncertainty, the observer need only increase the non-inductive resistance and the voltage of the circuit,[6] or the period of the galvanometer. The time of growth of a current in a circuit is proportional to the inductance divided by the resistance. To take one example of the time of growth of a current: a measurement was made of the field in a Helmholtz coil of 200 turns, 20 cm in diameter. The resistance of the coil was very small. Its inductance was about 12 millihenries. It was calculated that in order that the current should grow to within 1/2% of its final value in 0.005 of a second, a resistance of about 13 ohms should be in the circuit. Any slight deflection in so short a time would not be observed.

On account of sparking at the switch on breaking the circuit, when dealing with a very large coil or electromagnet, the null method may be better applied by leaving the fields on and suddenly removing or rotating the exploring coil. When no deflection occurs the fields are equal.

FAYERWEATHER LABORATORY OF PHYSICS,
AMHERST COLLEGE, AMHERST, MASS.

[6] Starling, Elec. and Mag., p. 304; 1924.

APPARATUS AND METHODS FOR PREPARING VERY CLEAN MERCURY

By Duane Roller

Any process which proves adequate for the purifying of a metal used as the active substance in a photoelectric cell can also be employed with considerable confidence in other fields of experimentation where extraordinary purity is desired. This follows from the fact that the highest obtainable degree of purity is the most essential requirement in the investigation of the photoelectric properties of a substance.

The following technique for cleaning mercury for photoelectric purposes has been used by the author with success[1]: (1) Remove grease by washing the mercury thoroughly with a 10 per cent solution of potassium hydroxide. (2) Shake or mix the mercury for several hours with an equal volume of 3-normal nitric acid, thus removing most of the metallic contamination. (3) Wash the mercury with distilled water, dry by heating it to about 110° C in a porcelain dish, and filter by forcing it through chamois skin, or by passing it through a pinhole in the apex of a glazed paper funnel. (4) Distill the mercury several times by the method of Hulett and Minchin;[2] in this method, air is bubbled through the mercury in the still during distillation, thus oxidizing volatile foreign metals. (5) Pass the mercury through a filter consisting of a glass tube with one end drawn to a capillary opening; repeat this process until all traces of oxide have disappeared from the surface of the mercury. (6) If it is desired to remove occluded gases, as is necessary in photoelectric work, distill the mercury repeatedly in the highest obtainable vacuum. The still used in this process should form an integral part of the apparatus in which the mercury finally is to be used. This method of removing occluded gases from mercury was first used in photoelectric work by Kazda.[3]

The washing processes described in steps (1), (2), and (3), above, are best accomplished with a centrifugal mixer (Fig. 1). This device can be made easily and it is more effective than the "fall tube," commonly employed in cleaning mercury, or than the ordinary mechanical

[1] Phys. Rev., *32*, p. 323; 1928.
[2] Phys. Rev., *21*, p. 388; 1905.
[3] Phys. Rev., *26*, p. 643; 1925.

stirrer; Bray[4] used a centrifugal mixer in preparing mercury, but does not describe it. This mixer is more efficient than the ordinary stirrer and undoubtedly can be put to many uses besides the present one, particularly in the organic chemistry laboratory. The vertical shaft of the mixer consists of a 30 cm length of 7 mm glass tubing. The upper end of the shaft is closed and to the lower end is sealed a 10 cm length of similar tubing, shaped as in Fig. 1, and drawn out at each end to an opening approximately 2 mm in diameter. This tube contains a third opening in its lower center, opposite the point where it is sealed to the shaft. The upper end of the shaft is fitted with a piece of short glass

FIG. 1. *Centrifugal mixer.*

tubing of slightly larger diameter than the shaft, this being held in place by two perforated corks which fit the shaft tightly. It is necessary to lubricate the part of the shaft between the two corks with a small amount of vaseline or graphite.

A clamp is attached to the larger tube, between the corks, and the mixer is supported vertically in a battery jar containing the mercury and washing liquid. The liquid should cover the openings in the ends of the mixer and the shaft should be so adjusted that air and mercury alternately enter the center opening during rotation. It is essential

[4] Am. Chem. Soc. J., *49*, p. 2572; 1927.

that the mixer be operated at high speed, preferably by means of a small motor attached to the shaft with rubber tubing or with a belt.

The still used in step (4) of the cleaning process should be constructed in the manner described by Hulett and Minchin, except that all joints made with rubber tubing or stoppers should be replaced by ground glass joints. Experience also shows that a receiving bottle for the distillate, connected to the condensing tube by a ground glass joint, is much more convenient and also less fragile than the usual tube of barometric height.

Since phosphorus pentoxide is often used as a drying agent in apparatus containing mercury, it might be well to state here that the phosphorous and lower oxides, often present in impure phosphorous pentoxide, are likely to soil mercury. Phosphorous pentoxide can be purified sufficiently for this purpose by heating it thoroughly in a current of dry air.

NOVEMBER, 1928,
UNIVERSITY OF OKLAHOMA,
NORMAN, OKLAHOMA.

APPARATUS FOR STUDYING NEWTON'S SECOND LAW

By Peter L. Tea

ABSTRACT

Attachments to a Free Fall Apparatus, described in a previous paper,* were developed for the use of students, to study Newton's Second Law experimentally, for: (1) pure translation, (2) pure rotation, and (3) combined translation and rotation.

TRANSLATION

The translation apparatus is shown in perspective in Fig. 1. The carriage C is equipped with two aluminum shell wheels, on which C rolls down the straight tubing A, under an unbalanced force f, equal

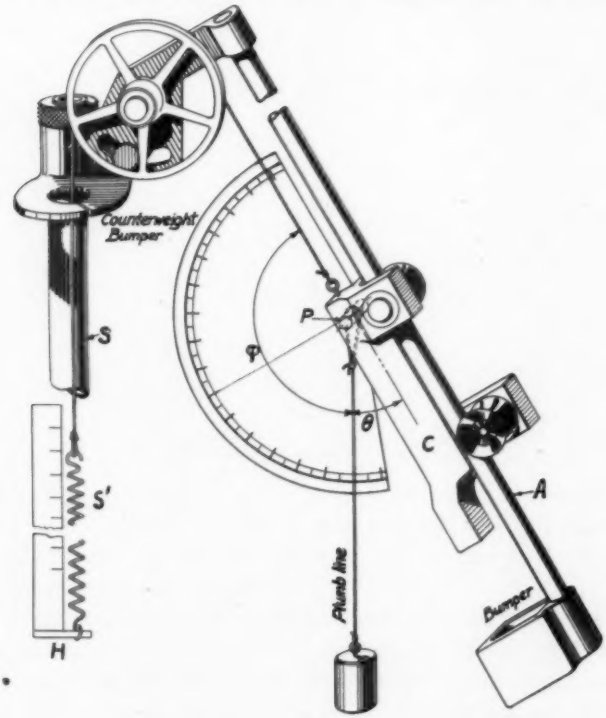


Fig. 1. *Perspective of translation apparatus.*

* J.O.S.A. & R.S.I., *17*, pp. 138–144; 1928.

to the component of the weight of C along A, less the combined resistance to motion. Crumpled paper, placed in the bumper, effectively reduces the shock. A helical spring, S', of steel wire, elongating 23.5 inches per lb, was used to measure the force f, producing the acceleration. The combination of ruler and one end of the spring, at H, was pulled down slowly, and the length of the spring read as soon as C moved up. The force f_1, corresponding to the stretch, equals the component of the weight of C, along A, plus the combined friction to motion; by slowly moving H up, until C just starts to roll down A, the stretch from which f was calculated was obtained. If the friction is independent of the speed, f *is* the force producing the acceleration. The air resistance was found to be negligible, for a 3'' dia. cardboard disc screwed under the weight W in the free fall machine, for a drop of 3.2 ft.

Fig. 1 of our previous paper, is a perspective of the free fall machine. The stand was removed and S of the translation attachment substituted. The cord attached to C goes over the pulley, and is connected to the cord drum. A record is made, such as Fig. 2, of equal distance intervals traversed by C, and the corresponding time, *from starting point*. The point of penetration of the pointed steel stylus in

TABLE 1. *Uniform acceleration in translation. Records 1, 3, 5, and 7 for weight of C=0.636 lbs.; records 2, 4, 6, and 8 for weight of C=0.788, and same θ sequence.*

Record	W lbs.	θ^0	f lbs.	f_1	a	f/a	W/g
No. 1	.636	53	0.367	0.394	17.7	.0207	.0198
3		46	.440	.461	21.6	.0204	
5		32.5	.503	.540	24.4	.0206	
7		20	.565	.596	29.1	.0194	
2	.788	53	.456	.493	17.7	.0258	.0245
4		46	.535	.571	21.3	.0251	
6		32.5	.629	.671	24.5	.0256	
8		20	.723	.743	29.4	.0246	

the record paper at the printed circumference, marks the instant of time corresponding to the number of the slit. The stylus cuts through to the edge of the paper, necessarily; but the slit thus made is merely incidental. It helps, by its greater length for the zero point, to pick this point out when the record extends more than one circumference.

Duplicate records superpose within 1/32,'' corresponding to .001 sec., for each point.

The angle θ, of A to the vertical, was obtained for each of the four positions used. The plumb bob and line, and the folded record, used to measure the angle, were removed for the actual tests. The angle θ, is given merely so that the reader can make other calculations from the data in Table 1.

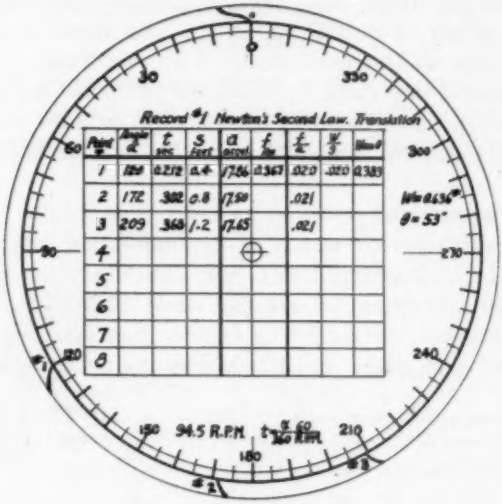


FIG. 2. *Record No. 1 of Table 1.*

Table 1 summarizes the results obtained for pure translation. The acceleration for each point, $s=0.4$, 0.8, and 1.2 ft., was calculated from $a=2s/t.^2$ This is justified because nearly the same values for a were obtained for the three points. In Table 1, the average values of a are given; see Fig. 2, which shows record No. 1, of Table 1 with complete data.

From Table 1, the student infers that for a given weight of C, and for any angle θ (and hence for any corresponding force):

$$f/a=\text{constant}. \tag{1}$$

For $\theta=0$, we have the special case of free fall, and the data show, as was to be expected, that

$$f/a=w/g \tag{2}$$

Some student will bring up the fact that both the pulley and the cord drum were accelerated at the same value as C, and that we did not take this into account. To answer him, and at the same time lead up to rotational inertia, we can *load* the apparatus with an extra pulley and an extra cord drum. Repeating the test will show an average decrease in a of about 3%, depending on the angle θ. We can also point out, in Table 1, that very nearly the same value for a was obtained, for the same θ, and different weights of C.

ROTATION

A lead disc was used, Fig. 3 (1); the torque can be applied either by a spring S, which should be protected by a guard, to prevent accident, or by weight w. Table 2 summarizes the results obtained, using

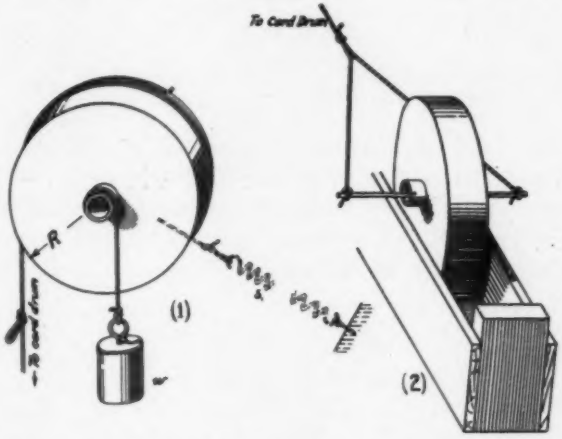


FIG. 3. *Solid disc of lead: (1) in pure rotation, (2) in both rotation and translation.*

TABLE 2. *Uniform acceleration in rotation, records 9, and 10; combined rotation and translation, records 11 and 12.*

Record	W lbs.	θ^o	f lbs.	f_1	a	f/a	W_o
No. 9	1.5		0.198	0.200	8.44	.0235	0.76
10			.320	.326	13.70	.0234	0.75
11		53.5	.873	.901	12.38	.0705	2.27
12		40	1.113	1.150	15.90	.0702	2.26

weight w. The operation is the same as before, except that a piece of match stick is tied to the cord, serving to prevent a sudden yank on the cord drum, by striking the counterweight bumper.

On applying eq. 2 to the results obtained, the equivalent weight W_e, considered balanced, and concentrated in the cord, was found to be very nearly one half the actual. On rolling the disc down an inclined plane, Fig. 3 (2), the equivalent weight was found to be equal to the sum of the actual plus the equivalent rotational.

The student can apply eq. 2 to each particle of the rotating disc problem, and show that the results obtained in Table 2 were to be expected. He thus infers that Newton's Second Law is fundamental in dynamics.

$$\frac{}{a} = \frac{W_e}{g} = \frac{1}{R^2}\int r^2 dm = \frac{I}{R^2} \tag{3}$$

$$\frac{\text{Torque}}{d^2\phi/dt^2} = I. \tag{4}$$

Equations 3, and 4 have the same form as eq. 2. The radius R in 3 may have any value with corresponding meaning of f, a and W_e.

Due credit is given to Mr. Phillip H. Courrier, for making the models used; his skill and long experience as a designer of instruments served also to improve the free fall apparatus.

COLLEGE OF THE CITY OF NEW YORK,
NEW YORK, N. Y.